10 Locally Covariant Approach to Effective Quantum Gravity

10 有效量子引力的局部协变方法

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Abstract

摘要

Despite the fact that quantum gravity is non-renormalizable, a consistent and mathematically rigorous construction of a perturbation series is possible. This is based on the use of the Batalin-Vilkovisky-Becchi-Rouet-Stora-Tyutin formalism for gauge theories, the methods of perturbative algebraic quantum field theory, and the principle of local covariance. The truncation of the series can be interpreted as an effective quantum field theory which provides predictions for observations at sufficiently small energy scales. Quantum cosmology can be seen as its lowest order expansion, and precision measurements on the cosmic microwave background yield the first empirical test of this approach to quantum gravity.

尽管量子引力是不可重整化的，但仍可对其微扰级数进行一致且数学上严格的构造。该构造基于规范理论的巴塔林-维尔可维斯基-贝基-鲁埃-斯托拉-秋京 (Batalin-Vilkovisky-Becchi-Rouet-Stora-Tyutin) 形式化、微扰代数量子场论方法，以及局部协变原理。截断后的级数可诠释为有效量子场论，能够为足够低能标下的观测给出预言。量子宇宙学可看作该理论的最低阶展开，对宇宙微波背景的精密测量已经为这种量子引力研究方法提供了首次实证检验。

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Keywords

关键词

Perturbative algebraic quantum field theory . Epstein-Glaser renormalization . relational observables · microwave background · Batalin-Vilkovisky formalism · BRST operator

微扰代数量子场论。爱波斯坦-格拉泽正规化。关系可观测量 · 微波背景 · 巴塔林-维尔可维斯基形式化 · BRST 算符

Introduction

引言

Fundamental physics at small scales is successfully described by the standard model of particle physics [21]. This is a quantum field theory (QFT) model, where the basic objects are quantum fields defined as operator-valued distributions on Minkowski space [34]. Effects of external gravitational fields can be included by generalizing the model to Lorentzian spacetimes; provided these spacetimes are globally hyperbolic, a consistent framework has been developed. This is locally covariant quantum field theory [5, 27] . The short distance problems caused by the curvature of the underlying spacetime are nowadays well understood; the interpretation of states, however, is not yet so clear. The reason is that the interpretation of states of QFT on Minkowski space is mainly done in terms of particles, but there is no known good concept of particles on generic spacetimes, a fact which is visible in some approaches as "particle creation by curvature" [29]. As a consequence, an experimental confirmation of the generalization to QFT on curved backgrounds is not yet available. Nevertheless, there are some predictions of this framework, in particular the radiation of black holes discovered by Hawking [25]. The latter prediction seems to be rather convincing [16], but a direct observation for presently known black holes is not possible, since the derived temperature of this radiation is too small. The expected existence of this radiation, however, can be considered as a hint for the way gravity is connected to quantum physics. Actually, it gave already rise to a rich and somewhat speculative literature (see, e.g., [26] and related papers).

小尺度下的基础物理可以由粒子物理标准模型成功描述 [21]。这是一个量子场论 (QFT) 模型，其基础对象是定义在闵氏空间上的算符值分布量子场 [34]。若要纳入外引力场的效应，可将该模型推广至洛伦兹时空；只要这些时空是整体双曲的，就已经构建出了自洽的框架，即局域协变量子场论 [5, 27] 。基础时空曲率引发的短程问题如今已经得到了充分理解，但量子态的解释仍不够清晰。原因在于，闵氏空间上 QFT 量子态的解释主要建立在粒子概念之上，而一般时空中并不存在成熟的粒子概念，这一点在“曲率产生粒子”这类表述中就有所体现 [29]。因此，将 QFT 推广到弯曲背景这一结论目前还没有得到实验验证。尽管如此，这个框架已经给出了一些预言，其中最著名的就是霍金发现的黑洞辐射 [25]。这一预言可信度很高 [16]，但由于该辐射的推导温度极低，目前无法对已知黑洞进行直接观测。不过，辐射存在的这一预期可以为引力与量子物理的关联提供线索。实际上，相关研究已经催生了大量颇具推测性的文献 (例如参见 [26] 及相关论文)。

In general, at typical scales of present physics, the quantum effects of the interaction of gravity with other fields seem to be rather small such that their neglection does not lead to conflicts with observation. Since gravity is very well described by general relativity which is a classical field theory of the spacetime metric, the direct way of unifying gravity with quantum physics is to add the spacetime metric as an additional field to the fields of the standard model. As a classical field theory this is well defined [17]. As a quantum field theory, however, severe problems occur.

一般而言，在当前物理学的典型能标下，引力与其他场相互作用的量子效应非常微弱，忽略这些效应并不会与观测结果产生矛盾。由于广义相对论作为时空度量的经典场论，已经能非常出色地描述引力，因此统一引力与量子物理最直接的方式，就是将时空度量作为额外场加入标准模型的场中。作为经典场论，这套框架是自洽定义好的 [17]。但作为量子场论，它存在严重的问题。

The main approach to the standard model is via perturbation theory. So one first builds a theory of non-interacting fields, where the dynamics is governed by a Lagrangian which is a quadratic functional of the basic fields. The higher-order terms of the Lagrangian are then treated as perturbations, and after several decades of hard work, the expansion could be constructed in terms of a formal power series where the lower orders could be explicitly calculated and yield a good and often excellent agreement with experiments. Crucial for this expansion is that there is only a finite number of parameters (around 25) which have to be determined by experiments so that predictions are possible, a fact which is due to the renormalizability of the model.

标准模型的主流研究方法是微扰论。我们首先构建非相互作用场的理论，其动力学由拉格朗日量描述，而拉格朗日量是基础场的二次泛函。随后将拉格朗日量的高阶项视为微扰，经过数十年的研究，这套展开可以表示为形式幂级数，低阶项可以显式计算，且结果与实验符合得很好，甚至非常出色。这套展开的关键之处在于，模型只存在有限个 (约 25 个) 需要由实验确定的参数，因此可以做出预言，这源于该模型是可重整的。

If one tries a corresponding approach to gravity, one has to choose the noninteracting theory, for instance the metric of Minkowski space with a theory of massless particles with helicity 2 (”gravitons”), and uses the difference to the full Einstein-Hilbert Lagrangian as an interaction. This interaction is, however, not a polynomial in the gravitational field and its derivatives. Since in four spacetime dimensions non-polynomial functionals of the free fields are not well defined (all relatively local fields are Wick polynomials, [2,14]), one replaces the interaction by a formal power series of polynomials. But then, it turns out that in the construction of the perturbative expansion, in every order new free parameters occur, so that strictly speaking no predictions are possible [22,28].

如果对引力尝试类似的方案，我们需要先选取非相互作用理论，例如闵氏空间度量加上螺旋度为 2 的无质量粒子 (即“引力子”) 理论，再将其与完整爱因斯坦-希尔伯特拉格朗日量的差值作为相互作用项。但该相互作用项并不是引力场及其导数的多项式。在四维时空中，自由场的非多项式泛函无法良好定义 (所有相对局域场都是维克多项式，参见 [2,14])，因此我们会将相互作用项替换为多项式构成的形式幂级数。但随后会发现，在构建微扰展开的过程中，每一阶都会出现新的自由参数，因此严格来说无法做出预言 [22,28]。

But the latter statement is too pessimistic. Namely, these free parameters have mass dimensions which are increasing with the order of the perturbative expansion. Hence at sufficiently small scales their influence is negligible. The fact that quantum effects of gravity are not easily visible indicates that our present observations take place at such scales.

但上述结论过于悲观。实际上，这些自由参数的质量维度会随着微扰展开的阶数升高而增加，因此在足够小的尺度下，它们的影响可以忽略。而引力量子效应难以观测这一事实本身就说明，我们目前的观测都处在这类尺度上。

The aim is therefore to ignore the problem of non-renormalizability (in the sense of the occurrence of a finite, nonzero number of free parameters in every order) and to develop a consistent theory of quantum gravity as a formal power series. After calculating the theory up to a given order, one can then determine the free parameters at this order and check whether the predictions agree with observations [11].

因此我们的目标是，暂且放下不可重整性的问题 (此处指每阶展开都会出现有限个非零自由参数的情况)，以形式幂级数的形式构建一套自洽的量子引力理论。将理论计算到给定阶数后，我们就可以确定该阶数的自由参数，再检验预言是否与观测吻合 [11]。

There are, however, further complications due to the necessity to satisfy the condition of general covariance. So the formalism should start from an arbitrary globally hyperbolic metric, but different choices have to lead to the same theory. Moreover, one has to define local observables, in spite of the fact that diffeomorphisms might change the localization region. Hence one has to build diffeomorphism-invariant combinations of the basic fields. In classical field theory this can be achieved by using some of the fields as coordinates and considering the remaining fields as functions of these coordinates (relational observables [8]; for a review see, e.g., [36]). It is by no means obvious how this can be done with quantum fields as operator-valued distributions. Moreover, the causal relations of the underlying spacetime are not stable against variations of the metric, but typically manifest themselves as algebraic relations between quantum fields.

然而，由于必须满足广义协变性条件，还存在进一步的复杂问题。因此该形式体系应当从任意整体双曲度规出发，但不同的选择必须得到同一个理论。此外，尽管微分同胚可能会改变定位区域，我们仍必须定义局部可观测量。因此必须构建基本场的微分同胚不变组合。在经典场论中，这可以通过将部分场用作坐标，并把其余场视为这些坐标的函数来实现 (关系可观测量 [8]；综述参见例如 [36])。如何将量子场作为算符值分布实现这一点完全不明确。此外，underlying 时空的因果关系无法在度规变化下保持稳定，反而通常体现为量子场之间的代数关系。

We will describe how a consistent framework for effective quantum gravity coupled to a scalar field can be formulated, and we show that in lowest nontrivial order it just coincides with the models used in quantum cosmology (see, e.g., [10]) to explain the fluctuations of the cosmic microwave background. This might be seen as a first visible quantum effect of gravity.

我们将阐述如何构建与标量场耦合的有效量子引力的自洽框架，并说明该框架在最低非平凡阶恰好与量子宇宙学中用来解释宇宙微波背景涨落的模型一致 (参见例如 [10])。这可以被看作引力第一个可观测的量子效应。

Relational Observables

关系可观测量

We consider a model of several scalar fields as a simplified version of the standard model coupled to the spacetime metric. Then we use scalar fields X_i which may be functions of the elementary fields and such that for generic values of the elementary fields the fields X_i form a coordinate system. In the quantized theory we expand the theory around such a background.

我们考虑一个由多个标量场构成的模型，作为耦合时空度量的标准模型的简化版本。随后我们采用标量场 X_i ，这些标量场可以是基本场的函数，且对于基本场的一般取值，X_i 构成一个坐标系。我们在量子化理论中围绕该背景展开理论。

Given a four-dimensional manifold M diffeomorphic to $\mathbb{R}^4$, we consider configurations $\Gamma = (g, \phi)$ where g is a Lorentz metric such that (M, g) is globally hyperbolic and ϕ is a smooth function with values in $\mathbb{R}^n$. The configurations are smooth sections of an affine bundle over M . Let $\mathcal{E}(M)$ denote the space of configurations Γ . This is an infinite dimensional affine manifold.

给定一个微分同胚于 $\mathbb{R}^4$ 的四维流形 M ，我们考虑构型 $\Gamma = (g, \phi)$: 其中 g 是使得 (M, g) 整体双曲的洛伦兹度量，ϕ 是取值于 $\mathbb{R}^n$ 的光滑函数。这些构型是 M 上仿射丛的光滑截面。设 $\mathcal{E}(M)$ 为构型 Γ 的空间，它是一个无限维仿射流形。

Diffeomorphisms χ of M act on these sections in a natural way, and this action extends uniquely to an action on the associated jet bundle and induces an action of the space $\mathcal{F}(M)$ of smooth functionals on $\mathcal{E}(M)$. We then consider functions $X_i, i = 1, \dots, 4$ on the jet bundle which transform as scalar fields, i.e., $\chi^* X_i(x) = X_i(\chi(x)), x \in M$. Examples of such functions are the scalar fields ϕ_i themselves and also suitable functionals of the metric, for instance the traces of powers of R_μ^ν , the Ricci tensor multiplied with the inverse metric. We now choose a configuration Γ_0 such that $x \mapsto X[\Gamma](x)$ is a diffeomorphism from M to $\mathbb{R}^4$ for Γ sufficiently near to Γ_0 . Let $\alpha[\Gamma]$ denote the diffeomorphism of M such that

M 的微分同胚 χ 以自然方式作用在这些截面上，该作用可唯一延拓到伴随射流丛上，并诱导出光滑函子空间 $\mathcal{F}(M)$ 在 $\mathcal{E}(M)$ 上的作用。随后我们考虑射流丛上变换为标量场的函数 $X_i, i = 1, \dots, 4$ ，即满足 $\chi^* X_i(x) = X_i(\chi(x)), x \in M$ 。这类函数的例子包括标量场 ϕ_i 本身，以及度量的合适函子，例如乘了逆度量的里奇张量 R_μ^ν 的幂次迹。现在我们选取一个构型 Γ_0 ，使得对于充分靠近 Γ_0 的 Γ ，$x \mapsto X[\Gamma](x)$ 是从 M 到 $\mathbb{R}^4$ 的微分同胚。设 $\alpha[\Gamma]$ 为 M 的微分同胚，满足

$$X[\Gamma] \circ \alpha[\Gamma] = X[\Gamma_0] \tag{1}$$

Given any tensor field $A[\Gamma]$, we define [8]

对于任意张量场 $A[\Gamma]$，我们定义 [8]

$$\mathcal{A}[\Gamma] = \alpha[\Gamma]^* A[\Gamma]$$

where $\alpha[\Gamma]^*$ is the pullback associated to $\alpha[\Gamma]$, and observe that $\mathcal{A}$ is invariant under diffeomorphisms, $\mathcal{A}[\chi^*\Gamma] = \mathcal{A}[\Gamma]$. Integrating $\mathcal{A}[\Gamma]$ with any test function, we obtain a smooth functional, i.e., an element of $\mathcal{F}(M)$.

其中 $\alpha[\Gamma]^*$ 是对应 $\alpha[\Gamma]$ 的拉回，可见 $\mathcal{A}$ 在微分同胚下不变，即 $\mathcal{A}[\chi^*\Gamma] = \mathcal{A}[\Gamma]$ 。将 $\mathcal{A}[\Gamma]$ 对任意测试函数积分，我们得到一个光滑函子，即 $\mathcal{F}(M)$ 中的一个元素。

We now expand $\mathcal{A}[\Gamma]$ around Γ_0 and obtain a formal power series of fields on the tangent space of the configuration space at the background Γ_0 . Up to first order this is

现在我们将 $\mathcal{A}[\Gamma]$ 在 Γ_0 附近展开，得到背景 Γ_0 处构型空间切空间上场的形式幂级数，一阶展开结果为

$$\mathcal{A}[\Gamma] = A[\Gamma_0] + \left\langle \frac{\delta A}{\delta \Gamma}[\Gamma_0], \Gamma - \Gamma_0 \right\rangle - \mathfrak{L}_Z A[\Gamma_0] \tag{2}$$

with the Lie derivative $\mathfrak{L}_Z$ for the vector field $Z = \left\langle \frac{\delta}{\delta\Gamma}\alpha[\Gamma_0], \Gamma - \Gamma_0 \right\rangle$. We see in particular that fields which vanish on the background configuration are at first order-invariant under diffeomorphisms, a fact which is known as the Stewart-Walker theorem [33].

其中包含对应向量场 $Z = \left\langle \frac{\delta}{\delta\Gamma}\alpha[\Gamma_0], \Gamma - \Gamma_0 \right\rangle$ 的李导数 $\mathfrak{L}_Z$ 。我们特别可以看到，在背景构型上为零的场在一阶下是微分同胚不变的，这就是已知的斯图尔特-沃克定理 [33]。

BV-BRST Formalism for Gravity

引力的 BV-BRST 形式主义

Main Ideas of the Formulation

表述的核心思想

As customary for gauge theories we describe the action of infinitesimal diffeomor-phisms by a fermionic vector field c (the ghost field) and the Becchi-Rouet-Stora-Tyutin (BRST) operator γ as the exterior derivative with respect to the action of diffeomorphisms on field configurations. More concretely, the action of γ on the metric and on the matter fields ϕ is given in terms of the Lie derivative, so

与规范理论的常规处理一致，我们通过费米矢量场 c (鬼场) 描述无穷小微分同胚的作用，并将贝奇-鲁埃-斯托拉-秋田 (BRST) 算子 γ 定义为对微分同胚在场构型上作用的外导数。更具体地说，γ 在度规和物质场 ϕ 上的作用由李导数给出，因此

$$\gamma g = \mathcal{L}_c g, \; \gamma\phi = \mathcal{L}_c \phi$$

where the ghost is treated as the evaluation functional on the space $\mathfrak{X}(M)$ of vector fields on M. For a functional $F \in \mathcal{F}(M)$, we have

> 其中鬼场被视为 M 上矢量场空间 $\mathfrak{X}(M)$ 的赋值泛函。对任意泛函 $F \in \mathcal{F}(M)$，我们有

$$\gamma F = \left\langle \frac{\delta F}{\delta g}, \mathcal{L}_c g \right\rangle + \left\langle \frac{\delta F}{\delta \phi}, \mathcal{L}_c \phi \right\rangle,$$

and for the ghost itself,

> 而对鬼场本身,

$$\gamma c = -\frac{1}{2}[c, c]$$

which is now an antisymmetric bilinear functional (bilinear form) on $\mathfrak{X}(M)$. We require γ to satisfy the graded Leibniz rule, which makes it a differential on the space of n-forms on $\mathfrak{X}(M)$, valued in functionals of the metric. It is convenient to think about antilinear forms on $\mathfrak{X}(M)$ as functionals of fermionic variables c^μ. Together with the field configurations Γ, the ghosts form a graded affine manifold that we denote $\overline{\mathcal{E}}(M)$ (extended configuration space). An element of $\overline{\mathcal{E}}(M)$ is a field multiplet φ, where the components φ^i run through all elementary fields of the model, i.e., the scalar fields ϕ_j, the components $g_{\mu\nu}$ of the spacetime metric, and the ghosts c^μ. We will keep denoting the space of functionals on $\overline{\mathcal{E}}(M)$ by $\mathcal{F}(M)$.

> 它现在是 $\mathfrak{X}(M)$ 上的反对称双线性泛函 (双线性型)。我们要求 γ 满足分次莱布尼茨规则，这使它成为取值在度规泛函空间、$\mathfrak{X}(M)$ 上 n-形式空间的微分。将 $\mathfrak{X}(M)$ 上的反线性形式视为费米子变量 c^μ 的泛函是很方便的。鬼场与场构型 Γ 共同构成一个分次仿射流形，我们将其记作 $\overline{\mathcal{E}}(M)$ (扩展构型空间)。$\overline{\mathcal{E}}(M)$ 的一个元素是场多重态 φ，其中分量 φ^i 涵盖模型的所有基本场，即标量场 ϕ_j 、时空度规的分量 $g_{\mu\nu}$ 和鬼场 c^μ 。我们仍将 $\overline{\mathcal{E}}(M)$ 上的泛函空间记作 $\mathcal{F}(M)$ 。

In order to include also the field equations, we extend the space of functionals to the space of vector fields $\mathcal{V}(M)$ on the extended configuration space which can formally be written as

> 为了同时纳入场方程，我们将泛函空间扩展为扩展构型空间上的矢量场空间 $\mathcal{V}(M)$，它可以形式地写为

$$\int X^i(x) \frac{\delta}{\delta \varphi^i(x)}.$$

The functional derivatives $\varphi_i^\ddagger \equiv \frac{\delta}{\delta \varphi^i}$ are called antifields and are treated as densities. Since we are dealing with graded quantities here, we need to specify whether we differentiate from the right or from the left. Unless stated otherwise, all the derivatives are left derivatives.

> 泛函导数 $\varphi_i^\ddagger \equiv \frac{\delta}{\delta \varphi^i}$ 被称为反场，按密度处理。由于我们这里处理的是分次量，需要指定是右微分还是左微分，除非另有说明，本文所有导数均为左导数。

It turns out to be convenient to embed field configurations and antifields in a graded manifold, where antifields have the opposite parity as the associated fields. The space of functions on that space is a graded commutative algebra. In this algebra one introduces an odd Poisson bracket $\{.,.\}$, the Schouten bracket, also called antibracket. For a vector field $X \in \mathcal{V}(M)$ and a functional $F \in \mathcal{F}(M)$ the bracket is just the application of the vector field to the functional,

将场构型和反场嵌入分次流形是十分方便的，在该流形中反场与对应场具有相反宇称。该流形上的函数空间是一个分次交换代数。我们在这个代数中引入一个奇泊松括号 $\{.,.\}$，即舒腾括号，也称为反括号。对矢量场 $X \in \mathcal{V}(M)$ 和泛函 $F \in \mathcal{F}(M)$ ，该括号就是矢量场对泛函的作用，

$$\{X, F\} = XF$$

and for two vector fields $X, Y \in \mathcal{V}(M)$ it coincides with the Lie bracket. For more general entries, it is extended by the graded Leibniz rule. This way we obtain a graded Poisson algebra $\mathcal{BV}(M)$, the Batalin-Vilkovisky (BV) algebra. See [7] for more details.

对两个矢量场 $X, Y \in \mathcal{V}(M)$ ，它与李括号一致。对更一般的元素，可通过分次莱布尼茨规则扩展得到。这样我们就得到了一个分次泊松代数 $\mathcal{BV}(M)$ ，即巴塔林-维尔可维斯基 (BV) 代数，更多细节参见文献 [7]。

The action S (for now we can think of it formally as a functional on $\mathcal{E}(M)$, the precise formulation follows in the next section) is the sum of the Einstein-Hilbert action and the action of the scalar fields which we assume to have the form

作用量 S (目前我们可以将它形式地看作 $\mathcal{E}(M)$ 上的泛函，精确表述将在下一节给出) 是爱因斯坦-希尔伯特作用量与标量场作用量之和，我们假设其形式为

$$\int \frac{1}{2} \sum_j d\phi_j \wedge * d\phi_j - V(\phi),$$

with the Hodge dual $*$ of the metric and an interaction density V . The field equations are obtained by the Schouten bracket of the action with the antifields and give rise to the Koszul-Tate differential

其中 $*$ 是度规的霍奇对偶，V 是相互作用密度。场方程可通过作用量与反场的舒腾括号得到，并引出柯苏尔-泰特微分

$$\delta(\bullet) = \{S, \bullet\}.$$

The action is invariant under diffeomorphisms and hence $\gamma S = 0$. Moreover, since γ is a graded derivation with respect to the Schouten bracket we have

该作用量在微分同胚下不变，因此满足 $\gamma S = 0$ 。此外，由于 γ 是关于舒滕括号的分次导子，我们有

$$(\delta\gamma + \gamma\delta)(\bullet) = \{S, \gamma(\bullet)\} + \gamma\{S, \bullet\} = \{\gamma(S), \bullet\} = 0.$$

Therefore, the classical BV operator,

因此，经典 BV 算符，

$$s = \gamma + \delta,$$

satisfies $s^2 = 0$. The cohomology of s then yields the gauge-invariant on-shell observables of the classical theory.

满足 $s^2 = 0$ 。s 的上同调给出经典理论的规范不变在壳可观测量。

We now expand γ in terms of antifields and obtain

现在我们将 γ 按反场展开，得到

$$\gamma(\bullet) = \left\{ \int \sum_i \gamma(\varphi^i)\varphi_i^\ddagger, \bullet \right\}$$

We can then write s as a Schouten bracket with the extended action

我们可以将 s 写为它与扩展作用量的舒滕括号

$$S_{\text{ext}} = S + \int \sum_i \gamma(\varphi^i)\varphi_i^\ddagger,$$

and the extended action satisfies the equation

且扩展作用量满足方程

$$\{S_{\text{ext}}, S_{\text{ext}}\} = 0, \tag{3}$$

called the classical master equation. We can express the classical BV operator as:

称为经典主方程。我们可以将经典 BV 算符表示为:

$$s = \{S_{\text{ext}}, \bullet\} .$$

In order to implement gauge fixing conditions, we extend $\overline{\mathcal{E}}(M)$ with additional scalar fields $b_j, \bar{c}_j, j = 0, \ldots, 3$. Consequently, $\mathcal{BV}(M)$ gets extended with the associated antifields and one needs to add extra terms to the action S_{ext} . The Nakanishi-Lautrup fields b_j are bosonic and transform as scalar fields under γ ,

为了施加规范固定条件，我们用额外标量场 $b_j, \bar{c}_j, j = 0, \ldots, 3$ 扩展 $\overline{\mathcal{E}}(M)$ 。相应地，$\mathcal{BV}(M)$ 会连带关联反场一起扩展，且需要给作用量 S_{ext} 添加额外项。中西-劳特鲁普场 b_j 是玻色场，在 γ 下按标量场变换,

$$\gamma(b_j) = c^v \partial_v b_j$$

The fields $\bar{c}_j$ (the antighosts) are fermionic and transform as

场 $\bar{c}_j$ (反鬼场) 是费米场，变换规则为

$$\gamma\left(\bar{c}_j\right) = -ib_j + c^{\upsilon}\partial_{\upsilon}\bar{c}_j. \tag{4}$$

Moreover, $\delta\left(\bar{c}_j\right) = \delta\left(b_j\right) = 0$. In this way the cohomology of s is not changed. The new extended action is still denoted by S_{ext} .

此外，$\delta\left(\bar{c}_j\right) = \delta\left(b_j\right) = 0$。通过这种构造，$s$ 的上同调不会发生改变。新的扩展作用量仍记为 S_{ext} 。

As gauge fixing fermion we use

我们选取以下规范固定费米子

$$\Psi = \int \sum_j i\bar{c}_j\left(d * dx_j - \frac{1}{2}b_j d\text{vol}\right),$$

with the canonical coordinates x_j of $\mathbb{R}^4$ and the density d vol induced by the metric. The new extended action is obtained by applying the canonical transformation of the graded Poisson algebra $\mathcal{BV}(M)$, which is induced by Ψ , i.e.:

其中 $\mathbb{R}^4$ 的正则坐标为 x_j ，d 是由度量诱导的体积密度。新的扩展作用量可以通过对由 Ψ 诱导的分次泊松代数 $\mathcal{BV}(M)$ 做正则变换得到，即:

$$\varphi^i \mapsto \varphi^i \,, \varphi_i^{\ddagger} \mapsto \varphi_i^{\ddagger} + \frac{\delta\Psi}{\delta\varphi^i}.$$

It is given in terms of the old one by

它通过旧作用量表示为

$$S_{\text{ext}} \mapsto S_{\text{ext}} + s(\Psi) = S_{\text{ext}} + \int \sum_j b_j\left(d * dx_j - \frac{1}{2}b_j d\text{vol}\right) - i\bar{c}_j d * dc^j$$

up to higher-order terms in the antifields.

展开到反场的高阶项之前成立。

The theory with the action in this form can be quantized, as the term with no antifields induces normally hyperbolic equations of motion. Different gauge fixings correspond to different canonical transformation of the original graded Poisson algebras and formally (i.e., under infinitesimal changes of the gauge fixing fermion) they are equivalent. In the final step, one sets antifields to zero and the resulting action is called the gauge-fixed action.

这种形式作用量对应的理论可以量子化，因为不含反场的项会导出正常双曲运动方程。不同的规范固定对应原始分次泊松代数不同的正则变换，且它们在形式上 (即规范固定费米子做无穷小变换时) 等价。最后一步将反场置零，得到的作用量称为规范固定作用量。

Precise Formulation

精确表述

One of the difficulties one faces when making all the ideas presented above precise is that nontrivial solutions to normally hyperbolic field equations on globally hyperbolic spacetimes cannot be compactly supported. This means that in order to have nontrivial on-shell theory, one cannot restrict oneself to compactly supported configurations. At the same time, globally hyperbolic spacetimes are necessarily non-compact, so the action S cannot be defined simply as the corresponding Lagrangian density $\mathcal{L}$ integrated over the whole M , and integration over compact subregions yields singular functionals. Instead, we use the approach of [6,9], where a generalized Lagrangian is a map L from the space of test functions $C_c^\infty(M)$ to the space of local functionals. This map has to be local and in mathematical terms this is expressed by the requirement that it is a natural transformation between certain functors. More concretely, given the usual Lagrangian density $\mathcal{L}(x)[\varphi]$ as a map on the jet bundle associated to the extended configuration space, one can define

将上述所有想法精确化时，我们面临的一个难点是: 整体双曲时空中正规双曲场方程的非平凡解无法具有紧支集。这意味着，要得到非平凡的壳上理论，我们不能将自身限制在紧支构型范围内。同时，整体双曲时空必然是非紧致的，因此作用量 S 不能简单定义为拉格朗日密度 $\mathcal{L}$ 在整个 M 上的积分，而在紧致子区域上积分又会得到奇异泛函。我们转而采用文献 [6,9] 中的方法: 广义拉格朗日是从测试函数空间 $C_c^\infty(M)$ 到局部泛函空间的映射 L 。该映射必须是局部的，在数学上这体现为它是特定函子之间的自然变换。具体来说，若已知通常的拉格朗日密度 $\mathcal{L}(x)[\varphi]$ 是扩张构型空间对应的射流丛上的映射，我们就可以定义

$$L(f) \doteq \int \mathcal{L}[f\varphi].$$

Actions are equivalence classes of Lagrangians under the equivalence relation

作用量是拉格朗日在如下等价关系下的等价类

$$L_1 \sim L_2 \text{ iff } \operatorname{supp}((L_1 - L_2)(f)) \subset \operatorname{supp}(df) \ ,$$

where the spacetime support of a functional $F \in \mathcal{F}(M)$ is defined by

其中泛函 $F \in \mathcal{F}(M)$ 的时空支集定义为

$$\operatorname{supp} F \doteq \{x \in M \mid \forall \mathcal{U} \ni x \text{ open neighborhood } \exists \varphi, \psi \in \overline{\mathcal{E}} \text{ with } \operatorname{supp}(\varphi - \psi) \subset \mathcal{U}$$

$$\text{s.t. } F(\psi) \neq F(\varphi)\}.$$

The classical master equation (3) is weakened to

经典主方程 (3) 被弱化为

$$\{S_{\text{ext}}, S_{\text{ext}}\} \sim 0,$$

where S_{ext} is the equivalence class corresponding to the generalized Lagrangian L_{ext}. We express the classical BV operator as

其中 S_{ext} 是对应广义拉格朗日 L_{ext} 的等价类。我们将经典 BV 算子表示为

$$sF = \{L_{\text{ext}}(f), F\},$$

where $f \equiv 1$ on supp F. We denote this operation by $\{S_{\text{ext}}, \bullet\}$.

其中 $f \equiv 1$ 定义在支集 F 上。我们将该运算记为 $\{S_{\text{ext}}, \bullet\}$。

Classical dynamics are implemented by means of the Euler-Lagrange derivative, which is a 1-form on $\overline{\mathcal{E}}(M)$ defined by:

经典动力学通过欧拉-拉格朗日导数实现，它是定义在 $\overline{\mathcal{E}}(M)$ 上的 1-形式，满足:

$$\langle dL(\varphi), \psi\rangle \doteq \left\langle \frac{\delta L(f)}{\delta\varphi}(\varphi), \psi \right\rangle,$$

where $f \equiv 1$ on support ψ. Here ψ is compactly supported $\left(\psi \in \overline{\mathcal{E}}_c(M)\right)$, so $\frac{\delta L(f)}{\delta\varphi}(\varphi) \in \overline{\mathcal{E}}_c'(M)$ is a distributional density without a restriction on support. For graded field configurations, $\frac{\delta}{\delta\omega^i}$ is the left derivative. Note that dL depends only on $S[L]$, the equivalence class of L.

其中 $f \equiv 1$ 定义在支集 ψ 上。此处 ψ 是紧支的 $\left(\psi \in \overline{\mathcal{E}}_c(M)\right)$，因此 $\frac{\delta L(f)}{\delta\varphi}(\varphi) \in \overline{\mathcal{E}}_c'(M)$ 是不限制支集的分布密度。对于分次场构型，$\frac{\delta}{\delta\omega^i}$ 是左导数。注意 dL 仅依赖于 $S[L]$，即 L 的等价类。

For the purpose of quantization, we will need a stronger version of this condition. Implementing this in practice for quantum gravity requires us to use not one, but two different test functions f_1, f_2, where f_1 is used for the Einstein-Hilbert and scalar fields Lagrangians and f_2 is used to multiply the gauge field c. This yields that the gauge transformations γ are compactly supported, and moreover we require that $f_1 \equiv 1$ on the support of f_2 since in this way the gauge transformations do not see the cutoff of the matter-metric part of the Lagrangian. Eventually, we define

为了量子化，我们需要该条件的一个更强版本。在量子引力中实际实现这一点要求我们使用两种而非一种测试函数 f_1, f_2 : 其中 f_1 用于爱因斯坦-希尔伯特拉格朗日量和标量场拉格朗日量，f_2 用于乘规范场 c。这可以保证规范变换 γ 是紧支的，此外我们要求 $f_1 \equiv 1$ 成立于 f_2 的支集上，因为这样规范变换就不会受到拉格朗日中物质-度规部分截断的影响。最终我们定义

$$L_{\text{ext}}\left(f_1,f_2\right)[\varphi,\varphi^{\ddagger}] \doteq \int \mathcal{L}\left(f_1 g, f_1\phi, f_1\bar{c}, f_1 b, f_2 c; \varphi^{\ddagger}\right)$$

and the antifields are also transformed, so that $\varphi_i^{\ddagger} \equiv \frac{\delta}{\delta(f_1\varphi^i)}$ for all i apart from the ghost indices for which we have $c^{\mu\ddagger} \doteq \frac{\delta}{\delta(f_2 c^{\mu})}$. With this definition, the gauge invariance of the original action implies that

且反场也会被变换，使得对除鬼指标外的所有 i 都有 $\varphi_i^{\ddagger} \equiv \frac{\delta}{\delta(f_1\varphi^i)}$ ，而鬼指标满足 $c^{\mu\ddagger} \doteq \frac{\delta}{\delta(f_2 c^{\mu})}$ 。根据该定义，原作用量的规范不变性可推出

$$\{L_{\text{ext}}\left(f_1,f_2\right), L_{\text{ext}}\left(f_1,f_2\right)\} = 0.$$

Perturbation Around a Background

背景下的微扰

We choose a background configuration $\Gamma_0 = (g_0, \phi_0)$ and $c = b = \bar{c} = 0$. The background configuration is chosen such that the dynamical coordinates $X_j\left[\Gamma_0\right]$ form a coordinate system. The background values of antifields are also set to zero. As a simple example we consider four scalar fields with $V = 0$ and use as background $\phi_{0_j} = x_j$ and a metric solving the Einstein equations with the energy momentum tensor $T_{\mu\nu}\left(\phi_0\right)$ such that also the field equation for ϕ_0 is satisfied. Then we can choose $X_j = \phi_j$ as dynamical coordinates.

我们选取一个背景构型 $\Gamma_0 = (g_0, \phi_0)$ 和 $c = b = \bar{c} = 0$ 。该背景构型需满足动力学坐标 $X_j\left[\Gamma_0\right]$ 构成一个坐标系。反场的背景值也设为零。我们举一个简单例子: 考虑四个标量场，满足 $V = 0$ ，选取 $\phi_{0_j} = x_j$ 作为背景，同时选取一个满足带能量动量张量 $T_{\mu\nu}\left(\phi_0\right)$ 的爱因斯坦方程的度规，使得 ϕ_0 的场方程也成立。此时我们可以选取 $X_j = \phi_j$ 作为动力学坐标。

We expand the generalized Lagrangian $L_{\text{ext}}\left(g_0 + \kappa h, \phi_0 + \kappa\varphi, \kappa c, \kappa b, \kappa\bar{c}, \kappa\varphi^{\ddagger}\right)$ in κ and obtain the decomposition

我们将广义拉格朗日量 $L_{\text{ext}}\left(g_0 + \kappa h, \phi_0 + \kappa\varphi, \kappa c, \kappa b, \kappa\bar{c}, \kappa\varphi^{\ddagger}\right)$ 按 κ 展开，得到分解式

$$L_{\text{ext}} = \kappa^2 L_0 + L_I(\kappa) + \text{ const}$$

where L_0 contains the terms of second order and L_I starts with terms of at least third order. Both L_0 and L_I can be expanded with respect to the antifield number, so that

其中 L_0 包含二阶项，L_I 从至少三阶项开始。L_0 和 L_I 都可以按反场数展开，因此有

$$L_0 = L_{00} + L_{01},\ L_I = L_{I0} + L_{I1}$$

For the chosen gauge fixing fermion, the Euler-Lagrange equation for L_{00} is Green hyperbolic [1], so the free theory can be constructed by means of deformation quantization and we construct the full theory as a formal power series in κ using perturbative algebraic quantum field theory (pAQFT) methods.

对于选定的规范固定费米子，L_{00} 的欧拉-拉格朗日方程是格林双曲型的 [1]，因此自由理论可以通过变形量子化构造，我们再利用微扰代数量子场论 (pAQFT) 方法将完整理论构造为 κ 中的形式幂级数。

The Feynman propagator obtains a factor κ^{-2} . The time-ordered powers of the interaction Lagrangian L_I are then formal power series in κ . The algebra of observables is now defined as the cohomology of the quantum BV operator, which is a deformation of the classical BV operator s . This contains the diffeomorphism-invariant formal power series obtained by expanding fields as functions of the scalar fields ϕ_j .

费曼传播子得到一个因子 κ^{-2} 。相互作用拉格朗日量 L_I 的时序乘积就是 κ 中的形式幂级数。可观测量代数现在定义为量子 BV 算子的上同调，这是经典 BV 算子 s 的变形。其中包含将场展开为标量场 ϕ_j 的函数得到的微分同胚不变形式幂级数。

Let us make these ideas more precise. We write the field equation for the generalized Lagrangian L_{00} in the form:

我们将这些想法具体化。我们将广义拉格朗日量 L_{00} 的场方程写为如下形式:

$$dL_{00}(\varphi) = P\varphi = 0,$$

where P is a Green hyperbolic operator. In terms of the components of the field multiplet φ , we have

其中 P 是格林双曲算子。用场多重态 φ 的分量表示，我们得到

$$P_{ij}(x) = \frac{\vec{\delta}}{\delta\varphi^i(x)} L_{00}(f) \frac{\overleftarrow{\delta}}{\delta\varphi^j(x)}, \tag{5}$$

with left and right derivatives, where $f \equiv 1$ on a compact neighborhood of x . Similarly

包含左右导数，其中 $f \equiv 1$ 定义在 x 的紧邻域上。同理可得

$$K^i_j(x) = \frac{\vec{\delta}}{\delta\varphi^{\ddagger}_j(x)} L_{01}(f) \frac{\overleftarrow{\delta}}{\delta\varphi^i(x)}. \tag{6}$$

We know that on globally hyperbolic spacetimes there exist retarded and advanced Green functions for P . We denote them by $\Delta^{\mathrm{R/A}}$. The Poisson bracket of the free theory is introduced using the Pauli-Jordan function

我们知道，在整体双曲时空中，P 存在推迟和超前格林函数，我们将它们记为 $\Delta^{\mathrm{R/A}}$ 。利用泡利-约当函数引入自由理论的泊松括号

$$\Delta \doteq \Delta^{\mathrm{R}} - \Delta^{\mathrm{A}}$$

and is defined by

其定义为

$$\lfloor F, G \rfloor \doteq \left\langle \frac{\vec{\delta} F}{\delta \varphi}, \Delta \frac{G \overleftarrow{\delta}}{\delta \varphi} \right\rangle,$$

where we suppressed all the indices. Here F and G are smooth functionals with smooth derivatives (i.e., $\frac{\delta F}{\delta \varphi}(\varphi), \frac{\delta G}{\delta \varphi}(\varphi)$ are smooth for all $\varphi \in \overline{\mathcal{E}}(M)$).

其中我们省略了所有指标。此处 F 和 G 是带光滑导数的光滑泛函 (即对所有 $\varphi \in \overline{\mathcal{E}}(M)$, $\frac{\delta F}{\delta \varphi}(\varphi), \frac{\delta G}{\delta \varphi}(\varphi)$ 都是光滑的)。

Note that this expression does not involve derivatives with respect to the antifields. To see that the bracket is also well-defined on the 0-th cohomology of $s_0 = \{S_0, \bullet\}$ (the space of gauge-invariant on-shell observables of the linearized theory), we need to check whether s_0 is a graded derivation with respect to the bracket $\lfloor ., . \rfloor$. We expand s_0 in the antifield number as $s_0 = \delta_0 + \gamma_0$, where $\delta_0 = \{S_{00}, \bullet\}$ implements the linearized equations of motion and $\gamma_0 = \{S_{01}, \bullet\}$ is the linearized BRST operator.

请注意，该表达式不涉及对反场的导数。要证明该括号在 $s_0 = \{S_0, \bullet\}$ 的零次上同调 (线性化理论的规范不变壳上可观测量空间) 上也是良定义的，我们需要验证 s_0 是否是关于括号 $\lfloor ., . \rfloor$ 的阶导子。我们将 s_0 按反场数展开为 $s_0 = \delta_0 + \gamma_0$ ，其中 $\delta_0 = \{S_{00}, \bullet\}$ 对应线性化运动方程，$\gamma_0 = \{S_{01}, \bullet\}$ 是线性化 BRST 算符。

For δ_0 , the argument is clear since Δ is itself a bisolution for P and $\delta_0 \varphi^\ddagger = P\varphi$. For γ_0 , we use the fact that the linearized field equations are invariant under the linearized BV operator γ_0 , so

对于 δ_0 ，论证很清晰，因为 Δ 本身就是 P 和 $\delta_0 \varphi^\ddagger = P\varphi$ 的双解。对于 γ_0 ，我们利用线性化场方程在线性化 BV 算符 γ_0 下不变这一性质，因此

$$K \Delta^{\mathrm{R}/A} + \Delta^{\mathrm{R}/A} K^\dagger = 0,$$

where $K^\dagger$ is the formal adjoint of K with respect to the pairing $\langle ., . \rangle$. The same goes for Δ . Consequently, γ_0 is a derivation with respect to $\lfloor ., . \rfloor$.

其中 $K^\dagger$ 是 K 相对于配对 $\langle ., . \rangle$ 的形式伴随。Δ 同理。因此，γ_0 是相对于 $\lfloor ., . \rfloor$ 的导子。

To deform this resulting Poisson algebra, we need to pick a state for the free (i.e., linearized) theory. We choose a quasifree Hadamard state with a two-point function Δ^+ . Being a Hadamard state means that Δ^+ is of positive type (i.e., $\Delta^+ \left(\bar{f}, f\right) \geq 0$, where f is a test function and $\bar{f}$ is its complex conjugate), the imaginary part of Δ^+ is given by $\frac{i}{2}\Delta, \Delta^+$ is a bisolution of P , and, crucially, Δ^+ satisfies the following wavefront set condition:

要形变这个得到的泊松代数，我们需要为自由 (即线性化) 理论选取一个态。我们选取一个两点函数为 Δ^+ 的拟自由哈达玛态。哈达玛态的性质是: Δ^+ 是正型的 (即 $\Delta^+ \left(\bar{f}, f\right) \geq 0$ ，其中 f 是测试函数，$\bar{f}$ 是其复共轭)，Δ^+ 的虚部由 $\frac{i}{2}\Delta, \Delta^+$ 给出，是 P 的双解，且至关重要的是，Δ^+ 满足以下波前集条件:

$$\mathrm{WF}(\Delta^+) = \left\{(x,k;x',-k') \in \dot{T}^*M^2 \mid (x,k) \sim (x',k'), k \in \left(\bar{V}_+\right)_x\right\},$$

where $(x,k) \sim (x',k')$ means that there exists a lightlike geodesic connecting the spacetime points x and x' with k and k' their respective cotangent vectors where the last one is the parallel transport of the former; moreover $\bar{V}_\pm$ is (the dual of) the closed future/past lightcone, seen as a subset of the cotangent bundle, and $\dot{T}^*M^2$ is the cotangent bundle deprived of its zero section. The wavefront set characterizes the singularity structure of Δ^+ and the above condition essentially says that we want to select states with the same singularity structure as the Minkowski vacuum. We write

> 其中 $(x,k) \sim (x',k')$ 表示存在类光测地线连接时空点 x 和 x'， k 和 k' 分别是两点对应的余切向量，且后者是前者的平行移动；此外 $\bar{V}_\pm$ 是余切丛子集意义下的闭未来/过去光锥 (的对偶)，$\dot{T}^*M^2$ 是去掉零截面的余切丛。波前集刻画了 Δ^+ 的奇异性结构，上述条件本质上说明我们要选取和闵氏真空奇异性结构相同的态。我们记

$$\Delta^+ = \frac{i}{2}\Delta + H$$

where H is the symmetric part, dependent on the choice of the Hadamard state. We define the star product corresponding to this choice as

> 其中 H 是对称部分，依赖于哈达玛态的选取。我们将对应该选取的星积定义为

$$(F \star G)(\varphi) = e^{\hbar\left\langle \frac{\vec{\delta}}{\delta\varphi_1}, \Delta^+ \frac{\vec{\delta}}{\delta\varphi_2}\right\rangle_F} F(\varphi_1)\, G(\varphi_2)\big|_{\varphi_1=\varphi_2=\varphi}.$$

Here F and G are smooth and have derivatives that satisfy wavefront set conditions that make them compatible with the wavefront set of Δ^+ . Basically, one can multiply two distributions, if their wavefront sets do not add up to a set which includes a zero section of the cotangent bundle. The precise condition is that

> 此处 F 和 G 光滑，且它们的导数满足的波前集条件使之与 Δ^+ 的波前集相容。基本结论是: 若两个分布的波前集相加后不包含余切丛的零截面，则这两个分布可相乘。精确条件为

$$\mathrm{WF}\left(F^{(n)}\left(\varphi,\varphi^\ddagger\right)\right) \subset \Xi_n,\ \forall n \in \mathbb{N}, \forall\varphi \in \overline{\mathcal{E}}(M), \tag{7}$$

where Ξ_n is an open cone defined as

> 其中 Ξ_n 是如下定义的开锥:

$$\Xi_n \doteq T^*M^n \setminus \left\{(x_1,\ldots,x_n;k_1,\ldots,k_n) \mid (k_1,\ldots,k_n) \in \left(\bar{V}_+^n \cup \bar{V}_-^n\right)_{(x_1,\ldots,x_n)}\right\}.$$

(8)

The same for G . Functionals satisfying this condition are called microcausal and we use the notation $\mathcal{BV}_{\mu c}(M)$ for this space of functionals. Equipped with the star product $\star$, this is the extended algebra associated to the free theory. For the star product to be compatible with s_0 , we also need to require that

G 同理。满足该条件的泛函称为微因果泛函，我们用 $\mathcal{BV}_{\mu c}(M)$ 标记该泛函空间。配备星积 $\star$ 后，这就是自由理论对应的扩展代数。要让星积与 s_0 相容，我们还需要要求

$$K\Delta^{+} + \Delta^{+}K^{\dagger} = 0,$$

i.e., that our Hadamard state is gauge invariant. With this extra requirement, $\star$ is well-defined on the cohomology of s_0 , which is now the space of gauge-invariant on-shell observables of free theory of the free quantum theory. Introducing the interaction is done using the methods of perturbative AQFT, as described in [6, 12, 31] . The interaction Lagrangian is L_I and after inserting a test function (or a pair of test functions, as described in section "Precise Formulation"), we obtain a compactly supported functional $V \equiv L_I(f_1, f_2)$.

也就是说，我们的阿达马态是规范不变的。在这一额外要求下，$\star$ 在 s_0 的上同调上是良定义的，该上同调现在是自由量子理论中自由理论的规范不变在壳可观测量空间。我们按照 [6, 12, 31] 中描述的微扰代数量子场论方法引入相互作用。相互作用拉格朗日量为 L_I ，插入测试函数 (或“精确表述”小节所述的一对测试函数) 后，我们得到一个紧支撑泛函 $V \equiv L_I(f_1, f_2)$ 。

Under these conditions, we can define interacting fields using the Bogoliubov formula. First we need the time-ordered products. Naively, time-ordered products would be defined in terms of the Feynman propagator,

在这些条件下，我们可以利用博戈留波夫公式定义相互作用场。我们首先需要得到时序积。朴素来说，时序积可以用费曼传播子定义，

$$\Delta^{F} = \frac{i}{2}\left(\Delta^{\mathrm{R}} + \Delta^{\mathrm{A}}\right) + H,$$

as

$$(F \cdot \mathcal{T} G)(\varphi) = e^{\hbar\left\langle \frac{\vec{\delta}}{\delta\varphi_1}, \Delta^{F} \frac{\vec{\delta}}{\delta\varphi_2}\right\rangle_F} F(\varphi_1)\, G(\varphi_2)\big|_{\varphi_1=\varphi_2=\varphi}.$$

This, however, makes sense only provided that F and G are regular functionals, i.e., all their derivatives are smooth. Since interesting observables are more singular than that, we need to employ (the extension of the) Epstein-Glaser renormalization [4, 15, 27] to extend the time-ordered product to more general arguments.

但这只有当 F 和 G 是正则泛函 (即它们的所有导数都是光滑的) 时才成立。由于我们感兴趣的可观测量比这更奇异，我们需要采用爱泼斯坦-格拉泽重整化 (的推广) [4, 15, 27] ，将时序积延拓到更一般的宗量。

One starts with defining the n -fold time-ordered products $\mathcal{T}_n(F_1, \dots F_n)$, where the functionals $F_i, i = 1, \dots n$ are formal power series with coefficients in local functionals. This includes the relational observables $\mathcal{A}[\Gamma]$ from section "Relational Observables" which can be expanded in a formal power series of local fields (again denoted by $\mathcal{A}[\Gamma]$) and smeared with a test density yield a sequence of local functionals, provided that the coordinates depend locally on the fields,

我们首先定义 n 重时序积 $\mathcal{T}_n(F_1, \dots F_n)$ ，其中泛函 $F_i, i = 1, \dots n$ 是系数为局部泛函的形式幂级数。这其中就包含“关系可观测量”小节的关系可观测量 $\mathcal{A}[\Gamma]$ ，这类可观测量可以展开为局域场的形式幂级数 (仍记为 $\mathcal{A}[\Gamma]$)，若坐标局部依赖于场，那么用测试密度抹平后就能得到一列局部泛函，

$$\mathcal{A}[\Gamma](f) \doteq \int \mathcal{A}[\Gamma](x) f(x) .$$

Epstein-Glaser renormalization allows one to construct each $\mathcal{T}_n$, assuming that all the lower-order k -fold time-ordered products ($k < n$) have been constructed and that they fulfill certain natural axioms. The most important axiom, which makes the procedure work, is the causal factorization property

> 爱泼斯坦-格拉泽重整化允许我们构造每一个 $\mathcal{T}_n$ ， 前提是所有低阶 k 重时序积 ($k < n$) 都已经构造完成，且它们满足若干自然公理。让这一过程成立的最重要公理是因果分解性质:

$$\mathcal{T}_n (F_1 \otimes \cdots \otimes F_n) = \mathcal{T}_k (F_1 \otimes \cdots \otimes F_k) \star \mathcal{T}_{n-k} (F_{k+1} \otimes \cdots \otimes F_n) ,$$

if the spacetime supports of $F_1, \ldots , F_k$ do not intersect the past of the spacetime supports of $F_{k+1}, \ldots , F_n$ (w.r.t. the background metric).

> 若 $F_1, \ldots , F_k$ 的时空支集不与 $F_{k+1}, \ldots , F_n$ 的时空支集的过去相交 (相对于背景度规而言)。

The Epstein-Glaser (EG) renormalization is a well-defined procedure if we work with formal power series in $\hbar$ and κ , even though the theory is power-counting non-renormalizable. However, the extensions obtained using the EG scheme are not unique and this nonuniqueness is described using the Stückelberg-Petermann renormalization group [6, 30, 35] . Roughly speaking, renormalization group transformations amount to adding finite counterterms, i.e., modifying the couplings. The difference between renormalizable and non-renormalizable theories is that in the former case, the total number of such parameters is finite and does not increase as we go to the higher orders in perturbation theory. This is not the case for gravity, which is power-counting non-renormalizable. However, we can still treat it as an effective theory, if we truncate the series at a finite order in $\hbar$ and κ .

> 即使该理论是幂次计数非可重整的，只要我们在 $\hbar$ 和 κ 下使用形式幂级数，爱泼斯坦-格拉泽 (EG) 重整化就是良定义的过程。但通过 EG 方案得到的延拓不唯一，这种非唯一性由施蒂克尔贝格-彼得曼重整化群 [6, 30, 35] 描述。粗略来说，重整化群变换等价于添加有限抵消项，即修改耦合常数。可重整理论和不可重整理论的区别在于: 可重整理论中，这类参数的总数是有限的，且不会随着微扰论阶数升高而增加。引力不满足这一点，它是幂次计数非可重整的。但只要我们将级数在 $\hbar$ 和 κ 的有限阶截断，就仍然可以将它作为有效理论处理。

With the n -fold time-ordered products at hand, we can define $\cdot_{\mathcal{T}}$ as a binary operation using the operator

> 得到 n 重时序积后，我们就可以利用算子将 $\cdot_{\mathcal{T}}$ 定义为一个二元运算:

$$\mathcal{T} \doteq \bigoplus_{n=0}^{\infty} \mathcal{T}_n \circ m^{-1}$$

where m^{-1} is the operation opposite to multiplication that allows one to factorize multilocal functionals into local ones [18]. This works also for products of relational observables constructed out of local fields, since they are expressed as power series in local functionals. Using $\mathcal{T}$, we deform the pointwise product $\cdot$ into the renormalized time-ordered product:

其中 m^{-1} 是与乘法相反的运算，可将多局部泛函分解为局部泛函 [18]。这也适用于由局域场构造的关系可观测量的乘积，因为这类可观测量本身就可以表示为局部泛函的幂级数。利用 $\mathcal{T}$，我们可以将逐点乘积 $\cdot$ 变形为重整化时序积:

$$F \cdot_{\mathcal{T}} G \doteq \mathcal{T}\left(\mathcal{T}^{-1}F \cdot \mathcal{T}^{-1}G\right).$$

One also deforms the classical BV operator of the free theory into

我们还需要将自由理论的经典 BV 算子变形为

$$\hat{s}_0 \doteq \mathcal{T}^{-1} \circ s_0 \circ \mathcal{T}$$

the quantum BV operator of the free theory. Applying $\mathcal{T}$ can be thought of as normal ordering, so given a classical observable $F, \mathcal{T}F$ is a corresponding free quantum observable.

自由理论的量子 BV 算符。应用 $\mathcal{T}$ 可以理解为正规序，因此给定经典可观测量 $F, \mathcal{T}F$，就得到了对应的自由量子可观测量。

Interaction is introduced by means of the quantum Møller maps. Let F be an observable of classical theory and $\mathcal{T}F$ the corresponding observable of the free quantum theory. We define the S-matrix associated with V by

相互作用通过量子缪勒映射引入。设 F 是经典理论的一个可观测量，$\mathcal{T}F$ 是对应的自由量子理论可观测量。我们定义与 V 关联的 S 矩阵为

$$\mathcal{S}(V) \doteq e_{\mathcal{T}}^{i\mathcal{T}V/\hbar} = \mathcal{T}e^{iV/\hbar}.$$

Using the Bogoliubov formula, we can now write down the quantum Møller operator R_V that allows us to construct interacting quantum observable corresponding to F :

利用博戈留波夫公式，我们现在可以写出量子缪勒算符 R_V，它可以用来构造对应于 F 的相互作用量子可观测量:

$$R_V(F) \doteq \mathcal{S}(\mathcal{T}V)^{-1} \star \left(\mathcal{S}(\mathcal{T}V) \cdot_{\mathcal{T}} \mathcal{T}F\right) = \left(\mathcal{T}e^{iV/\hbar}\right)^{-1} \star \mathcal{T}\left(e^{iV/\hbar} \cdot F\right).$$

As in the free case, we can deform the classical linearized BV operator to obtain the interacting quantum BV operator:

和自由场情形一样，我们可以对经典线性化 BV 算符做形变，得到相互作用量子 BV 算符:

$$\hat{s} \doteq R_V^{-1} \circ s_0 \circ R_V$$

This operator is a local operator, if we assume the quantum master equation (QME), which can be expressed as the condition:

如果我们假设量子主方程 (QME) 成立，该算符是局域算符，量子主方程可以表述为如下条件:

$$s_0 \mathcal{S}(V) = 0$$

This is equivalent to

这等价于

$$\frac{1}{2}\{L_{\text{ext}}(f_1, f_2), L_{\text{ext}}(f_1, f_2)\} - i\hbar \triangle_V = 0,$$

where $\triangle_V$ is the anomaly term that can be calculated using the anomalous Master Ward identity [3,18]. As explained in [7] we can use the renormalization freedom in defining the time-ordered products to ensure that the QME is fulfilled. Then the interacting quantum BV operator takes the form:

其中 $\triangle_V$ 是反常项，可以利用反常主沃德恒等式 [3,18] 计算。正如文献 [7] 所述，我们可以利用定义时序乘积时的重整化自由度来保证量子主方程成立。此时相互作用量子 BV 算符形式为:

$$\hat{s}F = sF - i\hbar \triangle_V F,$$

where $\triangle_V F \doteq \frac{d}{d\mu} \triangle_{V+\mu F}\Big|_{\mu=0}$ is the renormalized BV Laplacian. The cohomology of $\hat{s}$ describes the quantum gauge-invariant on-shell observables.

其中 $\triangle_V F \doteq \frac{d}{d\mu} \triangle_{V+\mu F}\Big|_{\mu=0}$ 是重整化后的 BV 拉普拉斯算符。$\hat{s}$ 的上同调描述了量子规范不变的在壳可观测量。

It remains to discuss the dependence of the chosen background and gauge fixing. The background independence can be shown by generalizing the formalism to backgrounds which are not solutions of the field equations. One then can show that a change of the background within a compact region induces a trivial automorphism of the algebra of gauge-invariant on-shell observables [7]. The independence of the choice of gauge fixing is intrinsic to the BV formalism, since a change of the gauge fixing fermion amounts to a symplectic transformation of the BV algebra which does not change the cohomology of the BV operator. Nevertheless, it would be necessary to show that this property is not lost by renormalization. (See the discussion in [37].)

最后还需要讨论对所选背景和规范固定的依赖。通过将形式化推广到非场方程解的背景，可以证明背景独立性。可以证明，紧致区域内的背景变换只会诱导规范不变在壳可观测量代数的平凡自同构 [7]。规范固定选择的独立性是 BV 形式体系固有的，因为改变规范固定费米子相当于对 BV 代数做辛变换，该变换不会改变 BV 算符的上同调。尽管如此，仍需要证明该性质在重整化后不会丢失。(参见文献 [37] 中的讨论。)

Cosmological Perturbation Theory

宇宙学微扰理论

We apply the general formalism to the case of gravity minimally coupled to a real scalar field (the dilaton) with self-interaction V [8]. As a background we choose a Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime with metric $g = a^2\eta$ where a is a function of the conformal time $\tau \equiv x^0$,

我们将该通用形式体系应用于引力与带自相互作用 V 的实标量场 (dilaton，dilaton 即 dilaton，这里保留原意: dilation 子) 最小耦合的情形 [8]。我们选取弗里德曼-勒梅特-罗伯逊-沃尔克 (FLRW) 时空作为背景，其度规为 $g = a^2\eta$ ，其中 a 是共形时间 $\tau \equiv x^0$ 的函数,

$$\eta \equiv \eta_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + \sum dx_i^2,$$

and as the background for the dilaton ϕ , we choose a function ϕ_0 depending only on τ . The equations of motions are satisfied, if a and ϕ_0 fulfill the equations

对于 dilation 子 ϕ 的背景，我们选取一个仅依赖于 τ 的函数 ϕ_0 。若 a 与 ϕ_0 满足下述方程，则运动方程成立:

$$(\phi_0')^2 + 2a^2 V(\phi_0) = 6\mathcal{H}^2$$

$$-(\phi_0')^2 + 2a^2 V(\phi_0) = 2\left(2\mathcal{H}' + \mathcal{H}^2\right) \tag{9}$$

$$\phi_0'' + 2\mathcal{H}\phi_0' + a^2 \frac{dV}{d\phi}(\phi_0) = 0$$

with $\mathcal{H} = a'/a$ and where $\bullet'$ denotes the derivative with respect to τ . $\mathcal{H}$ is related to the Hubble parameter H by $\mathcal{H} = Ha$ and to the Ricci scalar R by $R = 6\left(\mathcal{H}' + \mathcal{H}^2\right) a^{-2}$. Note that the third equation follows from the first two if ϕ_0' nowhere vanishes. We assume from now on that $\phi_0' < 0$.

其中 $\mathcal{H} = a'/a$ 为参数，$\bullet'$ 表示对 τ 的导数。$\mathcal{H}$ 与哈勃参数 H 满足关系 $\mathcal{H} = Ha$ ，与里奇标量 R 满足关系 $R = 6\left(\mathcal{H}' + \mathcal{H}^2\right) a^{-2}$ 。注意到若 ϕ_0' 处处非零，则第三个方程可由前两个方程推出。我们此后均假设 $\phi_0' < 0$ 成立。

A problem with this background is that due to its high symmetry there are not sufficiently many local functionals of the fields which can serve as coordinates. For one coordinate function we use the dilaton field ϕ itself and set

该背景存在一个问题: 由于其高度对称性,没有足够多可作为坐标的场的局域泛函。我们选取 dilation 子场 ϕ 本身作为其中一个坐标函数，并设置

$$X_0 = \phi_0^{-1} \circ \phi \equiv T \tag{10}$$

as our dynamical time function. The other coordinates are constructed as follows. We restrict the space-time metric to the hypersurfaces $\phi = \text{const}$. For ϕ near to ϕ_0 they are spacelike. We use the spatial coordinates x_i for the hypersurface given by $\phi = \text{const}$ and we will also need the following definition of the tangent fields at the hypersurface,

其为我们的动力学时间函数。其余坐标构造如下: 我们将时空度规限制在 ϕ = 为常数的超曲面上。当 ϕ 靠近 ϕ_0 时，这些超曲面是类空的。我们对 ϕ = 为常数的超曲面采用空间坐标 x_i，同时还需要如下超曲面切场的定义,

$$\partial_{i,\phi} = \partial_i - \frac{\partial_i \phi}{\partial_\tau \phi}\partial_\tau. \tag{11}$$

The components of the induced metric with respect to these coordinates are

诱导度量相对于这些坐标的分量为

$$g_{\phi,ij} \doteq g_\phi\left(\partial_{i,\phi}, \partial_{j,\phi}\right) = g_{ij} - g_{i0}\frac{\partial_j \phi}{\partial_\tau \phi} - g_{j0}\frac{\partial_i \phi}{\partial_\tau \phi} + g_{00}\frac{\partial_i \phi}{\partial_\tau \phi}\frac{\partial_j \phi}{\partial_\tau \phi}. \tag{12}$$

Let $\triangle_\phi$ denote the Laplacian with respect to the induced metric on this hypersurface (not to confuse with the graded BV Laplacian from the previous section). Explicitly, it is given by:

令 $\triangle_\phi$ 表示该超曲面上关于诱导度量的拉普拉斯算子 (请勿与上一节中分次 BV 拉普拉斯算子混淆)，其具体表达式为:

$$\triangle_\phi = \frac{1}{\sqrt{\det g_\phi}}\partial_{i,\phi} g_\phi^{ij}\sqrt{\det g_\phi}\partial_{j,\phi} = \frac{\partial_{i,\phi}\sqrt{\det g_\phi}}{\sqrt{\det g_\phi}} g_\phi^{ij}\partial_{j,\phi} + \partial_{i,\phi} g_\phi^{ij}\partial_{j,\phi}. \tag{13}$$

We then define

我们随后定义

$$X_i = \left(1 - G_\phi \Delta_\phi\right) x_i \tag{14}$$

where G_ϕ is the Green operator for $\triangle_\phi$ with vanishing boundary conditions at infinity.

其中 G_ϕ 是 $\triangle_\phi$ 满足无穷远零边界条件的格林算子。

The diffeomorphism α_Γ then assumes the following form.

微分同胚 α_Γ 则具有如下形式。

Proposition 1. Let Γ be a compactly supported variation of the background (i.e., supp $(\Gamma - \Gamma_0)$ is compact). Then

命题 1 令 Γ 是背景的紧支变分 (即 supp $(\Gamma - \Gamma_0)$ 是紧支的)，则

$$\alpha_\Gamma(\tau, x) = (\tau + \delta\tau, x + \delta x)$$

with

其中

$$\delta\tau = \phi(\bullet, x+\delta x)^{-1} \circ \phi_0(\tau) - \tau$$

and

且

$$\delta x_i = \frac{a^2}{4\pi}\int d^3y \frac{\triangle_\phi y^i}{|x-y|}.$$

Proof. On smooth functions $f \in \mathcal{C}^\infty(\mathbb{R}^3)$ we have

证明: 在光滑函数 $f \in \mathcal{C}^\infty(\mathbb{R}^3)$ 上我们有

$$\left(1 - G_\phi\left(\triangle_\phi - \triangle_{\phi_0}\right)\right)\left(1 + G_{\phi_0}\left(\triangle_\phi - \triangle_{\phi_0}\right)\right) f = f$$

where we use the resolvent equation

这里我们用到了预解式方程

$$G_\phi - G_{\phi_0} = G_\phi\left(\triangle_{\phi_0} - \triangle_\phi\right) G_{\phi_0},$$

which holds on compactly supported smooth functions.

该式在紧支撑光滑函数上成立。

The claim for δx_i now follows from the fact that $\triangle_{\phi_0}$ annihilates the coordinate functions and from the explicit form of G_{ϕ_0} . The claim for $\delta\tau$ is a simple consequence of the definition of T .

由 $\triangle_{\phi_0}$ 零化坐标函数这一性质及 G_{ϕ_0} 的显式形式，即可推得 δx_i 的结论。$\delta\tau$ 的结论是 T 定义的直接推论。

The proposition is expected to hold also for perturbations $\Gamma - \Gamma_0$ which are not compactly supported but vanish sufficiently fast at infinity. However, we refrain from entering this issue in the present review.

我们猜想该命题对非紧支撑但在无穷远处足够快衰减的微扰 $\Gamma - \Gamma_0$ 同样成立，但在本综述中我们不对此展开讨论。

Unfortunately, the chosen spatial coordinates are non-local. As a consequence, the associated invariant fields are invariant only under diffeomorphisms which tend sufficiently fast to the identity at infinity. This creates problems in higher-order perturbation theory where renormalization ambiguities for non-local functionals are not under control, in general. Up to first-order perturbation theory, however, this problem does not appear.

遗憾的是，所选空间坐标是非定域的，因此对应的不变场仅在无穷远处足够快趋近于恒等的微分同胚下保持不变。这会给高阶微扰理论带来问题: 一般而言，非定域泛函的重整化模糊性无法得到控制。不过该问题在一阶微扰论中不会出现。

In zeroth order, we have a free theory which coincides with the traditional cosmological perturbation theory [10], where the linearized classical equations hold also in the quantum theory. Quantum effects arise from the commutation relations and from the correlations in appropriate states.

零阶时我们得到的自由理论与传统宇宙学微扰论 [10] 一致，线性化经典运动方程在量子理论中依然成立，量子效应由对易关系和适当态中的关联产生。

The commutation relations are uniquely fixed by L_{00} . It is given by

对易关系由 L_{00} 唯一确定，其形式为

$$L_{00}(h,\varphi,c,b,\bar{c}) = \frac{1}{2}\langle (h,\varphi), \widetilde{P}(h,\varphi)\rangle + \langle (h,\varphi), Qb\rangle - \frac{1}{2}\sum_j \left(b_j^2 d\,\mathrm{vol} - i\bar{c}_j d * dc^j \right).$$

(15)

$\widetilde{P}$ and Q are differential operators depending on the background configuration Γ_0 . (See [24] for details, where P and $\widetilde{P}$ are interchanged and Q is denoted by K). The operator

$\widetilde{P}$ 和 Q 是依赖于背景构型 Γ_0 的微分算符。(详见文献 [24]，其中 P 和 $\widetilde{P}$ 互换，Q 记作 K)。该算符

$$P = \begin{pmatrix} \widetilde{P} & Q \\ Q^t & -1 \end{pmatrix} \tag{16}$$

is Green-hyperbolic [1], since $\widetilde{P} + QQ^t$ is normally hyperbolic. The advanced Green operator Δ^{A} for P is obtained from the advanced Green operator E_A of $\widetilde{P} + QQ^t$ by

是格林双曲算符 [1]，因为 $\widetilde{P} + QQ^t$ 是正规双曲算符。对应 P 的超前格林算符 Δ^{A} 可由 $\widetilde{P} + QQ^t$ 的超前格林算符 E_A 通过下式得到

$$\Delta^A = \begin{pmatrix} E_A & E_A Q \\ Q^t E_A & Q^t E_A Q \end{pmatrix}, \tag{17}$$

and an analogous formula holds for the retarded Green operator.

推迟格林算符满足类似的关系式。

The ghost term of the Lagrangian is decoupled at this order; hence it can be treated separately.

拉格朗日的鬼项在该阶退耦合，因此可以单独处理。

Let now Δ be the difference between the retarded and the advanced Green operator of P . The smeared fields

现在设 Δ 是 P 的推迟格林算符与超前格林算符之差，弥散场

$$A(f) = h(f_1) + \varphi(f_2) + b(f_3)$$

satisfy the commutation relations

满足对易关系

$$[A(f), A(g)] = i\langle f, \Delta g\rangle$$

and the field equation

以及场方程

$$A\left(P^t f\right) = 0.$$

Taking the generators $A(f)$ and the relations above, we obtain an algebra.

由生成元 $A(f)$ 和上述关系式，我们可以得到一个代数。

Another way to obtain this algebra, up to isomorphy, is to consider the algebra generated by the linear functionals of the form $A(f)$ and apply the procedure described in section ”BV-BRST Formalism for Gravity.” This amounts to taking the 0-homology of the Koszul-Tate operator, which then corresponds to taking the quotient by expressions of the form $A\left(P^t f\right)$.

在同构意义下, 得到该代数的另一种方法是: 考虑由形式为 $A(f)$ 的线性泛函生成的代数, 应用“引力的 BV-BRST 形式”一节所述的步骤，这等价于取 Koszul-Tate 算符的零同调，对应将形式为 $A\left(P^t f\right)$ 的表达式商去。

The ghosts and antighosts satisfy canonical anticommutation relations with the scalar commutator function multiplied by $-i$ and the corresponding field equation.

鬼场和反鬼场满足正则反对易关系，标量对易子函数乘以 $-i$，同时满足对应的场方程。

The linearized BRST transformation γ_0 maps linear fields to linear fields. It is given by

线性化 BRST 变换 γ_0 将线性场映射为线性场，其表达式为

$$\gamma_0\left(h_{\mu\nu}\right) = \nabla_\mu c_\nu + \nabla_\nu c_\mu$$

with the Levi-Civita connection ∇ of the background metric g_0,

其中用到背景度规 g_0 的列维-奇维塔联络 ∇，

$$\gamma_0(\varphi) = c^\mu \partial_\mu \phi_0, \gamma_0(\bar{c}_j) = -ib_j$$

and vanishes on the other fields.

且它在其余场上作用结果为零。

The fields which are at first-order gauge invariant are obtained by the expansion described in section "Relational Observables," Equation (2). In the case at hand, the vector field Z is given by

一阶规范不变场可通过“关联可观测量”一节的方程 (2) 中描述的展开得到。在本文讨论的情形中，矢量场 Z 的表达式为

$$Z = \frac{\varphi}{\phi_0'}\partial_\tau + Z^i \partial_i$$

with

其中

$$Z^i = \left\langle \frac{\delta X_i}{\delta \Gamma}[\Gamma_0], \Gamma - \Gamma_0 \right\rangle = G_0\left(\partial_i \frac{\mathcal{H}}{\phi_0'}\varphi + \partial_j h_i^j - \frac{1}{2}\partial_i h_j^j\right) \equiv X_i^{(1)}, \tag{18}$$

where G_0 is the Green operator of the Laplacian on $\mathbb{R}^3$ (i.e., the convolution with the Coulomb potential). To obtain the above formula, we used the fact that differentiating (13), one obtains

G_0 是 $\mathbb{R}^3$ 上拉普拉斯算子的格林算子 (即与库仑势的卷积)。为得到上述公式，我们利用了对 (13) 求导可得

$$\left\langle \frac{\delta \Delta_\phi}{\delta \phi}[a^2\eta, \phi_0], \varphi \right\rangle = -\frac{1}{a^2\phi_0'}\left(2(\partial_i\varphi)\partial_i\partial_\tau + \mathcal{H}(\partial_i\varphi)\partial_i + (\triangle\varphi)\partial_\tau\right)$$

and

以及

$$\left\langle \frac{\delta \triangle_\phi}{\delta g}[a^2\eta, \phi_0], h \right\rangle = \sum_{ij} a^{-4}\left(\frac{1}{2}(\partial_i h_{jj})\partial_i - \partial_i h_{ij}\partial_j\right).$$

Next, we find that

接下来，我们得到

$$\gamma_0(Z^\mu) = c^\mu. \tag{19}$$

Now we use formula (2), with A the metric and where $A[\Gamma_0]$ is $a^2\eta$, and obtain the gauge-invariant fields

现在我们使用公式 (2)，其中 A 为度规，$A[\Gamma_0]$ 为 $a^2\eta$ ，得到规范不变场

$$\tilde{g}_{\mu\nu} = a^2\eta_{\mu\nu} + h_{\mu\nu} - \partial_\mu Z_\nu - \partial_\nu Z_\mu + 2\Gamma_{\mu\nu}{}^{\lambda} Z_\lambda. \tag{20}$$

Using (19) we verify that $\gamma_0\left(\tilde{g}_{\mu\nu}\right) = 0$.

利用 (19) 我们验证得到 $\gamma_0\left(\tilde{g}_{\mu\nu}\right) = 0$ 。

The non-vanishing Christoffel symbols for the background metric $a^2\eta$ are

背景度规 $a^2\eta$ 的非零克里斯托费尔符号为

$$\Gamma_{00}{}^{0} = \Gamma_{ii}{}^{0} = \Gamma_{i0}{}^{i} = \Gamma_{0i}{}^{i} = \mathcal{H}, i = 1, 2, 3.$$

Defining $\widetilde{h} = \tilde{g} - a^2\eta$ and using the convention $Z_\mu = a^2\eta_{\mu\nu}Z^\nu$, we obtain

定义 $\widetilde{h} = \tilde{g} - a^2\eta$ 并采用约定 $Z_\mu = a^2\eta_{\mu\nu}Z^\nu$ ， 我们得到

$$\widetilde{h}_{00} = h_{00} - 2\partial_0 Z_0 + 2\mathcal{H}Z_0 = h_{00} + 2a^2\left(\partial_\tau + \mathcal{H}\right)\frac{\varphi}{\phi_0'}$$

$$\widetilde{h}_{0i} = h_{0i} - \partial_0 Z_i - \partial_i Z_0 + 2\mathcal{H}Z_i = h_{0i} - a^2\partial_\tau X_i^{(1)} + a^2\partial_i\frac{\varphi}{\phi_0'}$$

$$\widetilde{h}_{ij} = h_{ij} - \partial_i Z_j - \partial_j Z_i + 2\mathcal{H}\delta_{ij}Z_0 = h_{ij} - a^2\left(\partial_i X_j^{(1)} + \partial_j X_i^{(1)} + 2\mathcal{H}\delta_{ij}\frac{\varphi}{\phi_0'}\right).$$

All the fields other than $\widetilde{h}_{00}$ are non-local due to the occurrence of G_0 . However, local fields can be easily obtained by applying the Laplace operator to them. Since the dilaton field was used as a coordinate, the corresponding gauge-invariant field

除 $\widetilde{h}_{00}$ 外的所有场都是非定域的，因为出现了 G_0 。但我们可以很容易地通过对它们作用拉普拉斯算子得到定域场。由于 dilaton 场被用作坐标，对应的规范不变场

$$\widetilde{\phi} = \phi_0 + \varphi - Z^0\partial_\tau\phi_0 = \phi_0$$

is trivial. Using the formula (2) we also find that $\widetilde{X}_i^{(1)} = 0$. Now applying (18) we obtain the following constraints:

是平凡的。利用公式 (2) 我们还得到 $\widetilde{X}_i^{(1)} = 0$ 。现在应用 (18) 我们得到如下约束:

$$\sum_j \partial_j \widetilde{h}_{ji} = \sum_j \frac{1}{2}\partial_i \widetilde{h}_{jj}$$

so the fields $\widetilde{h}_{ij}$ are not independent.

因此场 $\widetilde{h}_{ij}$ 不是独立的。

It is customary to parametrize the metric perturbation h in terms of scalars, vector, and tensor fields with respect to the Euclidean symmetry of $\mathbb{R}^3$. Note however that this parametrization is unique only if all these fields vanish at infinity. Moreover, these fields are non-local functionals of the field configuration.

人们通常依据 $\mathbb{R}^3$ 的欧几里得对称性，将度规微扰 h 参数化为标量场、矢量场和张量场的组合。但需要注意，只有当所有这些场在无穷远处为零时，该参数化才是唯一的。此外，这些场是场构形的非定域泛函。

The space-space components of h in that parametrization are given by

该参数化中 h 的空间-空间分量由下式给出

$$h_{ij} = 2a^2\left(\partial_i\partial_j E + \delta_{ij} D + \partial_{(i} W_{j)} + T_{ij}\right)$$

with a tensor field T with $\sum_i T_{ii} = 0$ and $\sum_i \partial_i T_{ij} = 0$, a vector field W with $\sum_i \partial_i W_i = 0$, and scalar fields D and E with $D = \frac{1}{2}\left(\sum_j h_{jj} - \sum_{ij} G_0 \partial_i \partial_j h_{ij}\right)$ and $E = G_0\left(\frac{1}{2a^2}\sum_j h_{jj} - 3D\right)$. In terms of these fields, the first-order coordinate fields are

其中张量场 T 满足 $\sum_i T_{ii} = 0$ 和 $\sum_i \partial_i T_{ij} = 0$ ，矢量场 W 满足 $\sum_i \partial_i W_i = 0$ ，标量场 D 和 E 满足 $D = \frac{1}{2}\left(\sum_j h_{jj} - \sum_{ij} G_0 \partial_i \partial_j h_{ij}\right)$ 和 $E = G_0\left(\frac{1}{2a^2}\sum_j h_{jj} - 3D\right)$。用这些场表示，一阶坐标场为

$$X_i^{(1)} = \partial_i E + W_i + \frac{\mathcal{H}}{\phi_0'} G_0 \partial_i \mu$$

with the Mukhanov-Sasaki variable $\mu = \varphi - \frac{\mathcal{H}}{\phi_0'} D$. (Unfortunately, in [8], the field W was missing in the corresponding formula.)

其中引入了 Mukhanov-Sasaki 变量 $\mu = \varphi - \frac{\mathcal{H}}{\phi_0'} D$ 。(遗憾的是，文献 [8] 的对应公式中遗漏了场 W 。)

For the gauge-invariant fields $\widetilde{h}_{ij}$ we find

对于规范不变场 $\widetilde{h}_{ij}$ ，我们得到

$$\widetilde{h}_{ij} = 2a^2\left(T_{ij} + \frac{\mathcal{H}}{\phi_0'}\left(G_0 \partial_i \partial_j - \delta_{ij}\right)\mu\right).$$

The time-time component is $h_{00} = -2a^2 A$ with a scalar field A . The corresponding gauge-invariant field is

时间-时间分量为 $h_{00} = -2a^2 A$ ，其中 A 是标量场。对应的规范不变场为

$$\widetilde{h}_{00} = 2a^2\left(\left(\partial_\tau + \mathcal{H}\right)\frac{\varphi}{\phi_0'} - A\right).$$

The time-space component is written as

时间-空间分量可写为

$$h_{0i} = a^2 \left(V_i - \partial_i B\right)$$

with a scalar field B and a divergence free vector field V . For the gauge-invariant vector field $\widetilde{h}_0$. we get

其中 B 是标量场，V 是无散矢量场。对于规范不变矢量场 $\widetilde{h}_0$ ，我们得到

$$\widetilde{h}_{0i} = a^2 \left(\left(V - W'\right)_i + \frac{1}{\phi_0'} \partial_i \left(\chi - G_0 \mu'\right) \right)$$

with the vector field $V - W'$ and the scalar field $\chi = \varphi - \phi_0' \left(B + E'\right)$.

其中 $V - W'$ 是矢量场，$\chi = \varphi - \phi_0' \left(B + E'\right)$ 是标量场。

We observe that our gauge-invariant fields can be parametrized by the fields μ , $\chi, T, V - W'$, and $\Phi = A - \left(\partial_\tau + \mathcal{H}\right)\left(B + E'\right)$, which are the gauge-invariant fields traditionally used in cosmological perturbation theory. Moreover, these fields are uniquely determined by the fields $\widetilde{h}_{\mu\nu}$. Note, however, that the mentioned parametrization induces additional non-localities, which can give rise to spurious violations of causality [13]. On the contrary, the fields $\triangle \widetilde{h}_{\mu\nu}$ are local and have to satisfy the usual causal relations.

我们发现，我们的规范不变场可以由场 μ 、$\chi, T, V - W'$ 和 $\Phi = A - \left(\partial_\tau + \mathcal{H}\right)\left(B + E'\right)$ 参数化，这些正是宇宙扰动理论中传统使用的规范不变场。此外，这些场由场 $\widetilde{h}_{\mu\nu}$ 唯一确定。但需要注意的是，上述参数化会引入额外的非局域性，这可能导致虚假的因果性破坏 [13]。相反，场 $\triangle \widetilde{h}_{\mu\nu}$ 是局域的，满足常规因果关系。

This construction of gauge-invariant fields may be illustrated by relating it to geometrical objects. We use the tangent fields in (11) and compute the spatial curvature tensor at first order

这种规范不变场的构造可以通过关联到几何对象来说明。我们利用 (11) 中的切矢量场，计算一阶空间曲率张量

$$R^{\phi l}_{ijk} = \frac{1}{2a^2} \left(\partial_i \partial_k h_{jl} - \partial_j \partial_k h_{il} - \partial_i \partial_l h_{jk} + \partial_j \partial_l h_{ik}\right)$$

$$+ \frac{\mathcal{H}}{a \phi_0'} \left(\partial_i \partial_l \varphi \delta_{jk} - \partial_j \partial_l \varphi \delta_{ik} - \partial_i \partial_k \varphi \delta_{jl} + \partial_j \partial_k \varphi \delta_{il}\right)$$

$$= \frac{1}{2a^2} \left(\partial_i \partial_k \widetilde{h}_{jl} - \partial_j \partial_k \widetilde{h}_{il} - \partial_i \partial_l \widetilde{h}_{jk} + \partial_j \partial_l \widetilde{h}_{ik}\right).$$

It is already gauge invariant, in agreement with the Stewart-Walker theorem, since it vanishes on the background.

它已经是规范不变的，符合 Stewart-Walker 定理，因为它在背景上为零。

The extrinsic curvature, however,

然而，外曲率

$$K_{ij}^{\phi}=-N_{\phi}\left\langle d\phi,\nabla_{\partial_{i,\phi}}\partial_{j,\phi}\right\rangle,\ \text{with}\ N_{\phi}=\left|\mathrm{g}^{-1}\left(d\phi,d\phi\right)\right|^{-\frac{1}{2}},$$

assumes on the background the value

在背景上的取值为

$$K_{0\,ij}^{\phi}=a\mathcal{H}\delta_{ij}$$

and therefore its first-order contribution

因此它的一阶贡献

$$K_{1ij}^{\phi}=\frac{\mathcal{H}}{2a}h_{00}\delta_{ij}-\frac{a}{\phi_0'}\partial_i\partial_j\varphi$$

is not gauge invariant.

不是规范不变的。

A gauge-invariant first-order contribution $\widetilde{K}_1^{\phi}$ to the extrinsic curvature is obtained by evaluating it at an infinitesimally shifted conformal time $\tau'(\varphi,\tau)$, which amounts to replace h_{00} by $\widetilde{h}_{00}$ and φ by 0, i.e.,

外曲率的规范不变一阶贡献 $\widetilde{K}_1^{\phi}$ 可以通过在无穷小平移后的共形时间 $\tau'(\varphi,\tau)$ 处计算得到，这等价于将 h_{00} 替换为 $\widetilde{h}_{00}$ ，将 φ 替换为 0，即

$$\widetilde{K}_{1\,ij}^{\phi}=\frac{\mathcal{H}}{2a}\widetilde{h}_{00}\delta_{ij}.$$

Quantum Gravity and the Cosmic Microwave Background

量子引力与宇宙微波背景

The observed cosmic microwave background (CMB) can, to a large extent, be described by a state of the free electromagnetic field on an FLRW spacetime which satisfies the Kubo-Martin-Schwinger (KMS) condition [23] with respect to conformal time. This is a quasifree state with the two-point function

观测到的宇宙微波背景 (CMB) 在很大程度上可以用 FLRW 时空上满足共形时间下久保-马丁-施温格 (KMS) 条件 [23] 的自由电磁场态来描述。这是一个具有两点函数的准自由态

$$\omega_{\beta}\left(F_{\mu\nu}(x)F_{\rho\sigma}(y)\right)=(2\pi)^{-3}\int d^4p\,\delta\left(p^2\right)P_{\mu\nu\rho\sigma}(p)\,e^{i(x-y)p}\frac{\theta\left(p^0\right)+e^{\beta p}\theta\left(-p^0\right)}{1-e^{-\beta p}}$$

with

其中

$$P_{\mu\nu\rho\sigma} = p_\mu p_\rho \eta_{\upsilon\sigma} - p_\upsilon p_\rho \eta_{\mu\sigma} + p_\upsilon p_\sigma \eta_{\mu\rho} - p_\mu p_\sigma \eta_{\upsilon\rho}.$$

β is here a four-vector in the interior of the future light cone in Minkowski space, and we use the notation $yp = y^\mu p_\mu, p_\mu = \eta_{\mu\nu} p^\nu$, and $p^2 = -p_\mu p^\mu . \beta$ characterizes a rest system determined by the heat bath. Its proper time

β 是闵可夫斯基空间未来光锥内部的一个四矢量，我们采用记号 $yp = y^\mu p_\mu, p_\mu = \eta_{\mu\nu} p^\nu$ ，且 $p^2 = -p_\mu p^\mu . \beta$ 表征由热浴确定的静止系。它的固有时

$$|\beta| = \sqrt{\beta^2}$$

is the inverse of the temperature relative to conformal time. In cosmic time t , related to conformal time τ by $dt = a d\tau$, this state is not an equilibrium state. But since the scale parameter a varies slowly, one can interpret it approximately as an equilibrium state with time-dependent temperature $\mathfrak{T} = \frac{1}{a|\beta|}$. Its temperature at the time of recombination is related to the binding energy of hydrogen. This then yields the nowadays observed temperature.

是相对于共形时间的温度的倒数。在与共形时间 τ 满足关系 $dt = a d\tau$ 的宇宙时间 t 中，该态不是平衡态。但由于标度参数 a 变化缓慢，我们可以将近似将其解释为具有含时温度 $\mathfrak{T} = \frac{1}{a|\beta|}$ 的平衡态。复合时期的温度与氢的结合能相关，由此可得我们如今观测到的温度。

The electromagnetic radiation in the relevant frequencies does interact only weakly with the mostly neutral matter after recombination. The main deviations from the simple behavior as a conformal KMS state are due to variations of the metric. This is known as the Sachs-Wolfe effect [32]. Some of these variations of the metric are caused by inhomogeneities in the distribution of matter in the universe. But in addition one observes small fluctuations which can be interpreted as quantum fluctuations of the gravity-dilaton system described in the previous section.

相关频率的电磁辐射在复合之后仅与大部分呈电中性的物质发生弱相互作用。共形 KMS 态简单行为的主要偏差来源于度规的变化，这就是著名的萨克斯-沃尔夫效应 [32]。部分度规变化由宇宙中物质分布的不均匀性引发，但此外还观测到微小涨落，这些涨落可以被解释为上一节介绍的引力-伸缩子系统的量子涨落。

The observed electromagnetic field can be related to the field at the time τ_r of recombination by using the free Maxwell equations in a spacetime with metric $a^2\eta + h$. For this purpose we choose a smooth function χ of conformal time which is equal to 1 for $\tau < \tau_r - \varepsilon$ and vanishes for $\tau > \tau_r + \varepsilon$. The free Maxwell equation for the field strength F (considered as a 2-form) is

我们可以利用度规为 $a^2\eta + h$ 的时空中的自由麦克斯韦方程，将当前观测到的电磁场与复合时期 τ_r 的场联系起来。为此我们选取一个共形时间的光滑函数 χ ，它在 $\tau < \tau_r - \varepsilon$ 时等于 1，在 $\tau > \tau_r + \varepsilon$ 时等于 0。场强 F (视为 2-形式) 满足的自由麦克斯韦方程为

$$(d+\delta)F=0$$

with the codifferential δ . The differential operator $d+\delta$ is Green hyperbolic [1], i.e., it has unique retarded and advanced Green operators G_{ret} and G_{adv} . Let now f be a compactly supported 2-form with support contained in a neighborhood of our own spacetime position with $\tau > \tau_r + \varepsilon$ for $(\tau, x) \in \operatorname{supp} f$. We have

其中余微分是 δ 。微分算子 $d+\delta$ 是格林双曲型 [1]，即它具有唯一的推迟和超前格林算子 G_{ret} 与 G_{adv} 。设 f 是紧支集 2-形式，其支集包含在我们所在时空位置附近的邻域内，且满足 $\tau > \tau_r + \varepsilon$ 当 $(\tau, x) \in \operatorname{supp} f$ 。我们有

$$f=(d+\delta)\chi G_{\text{adv}} f+(d+\delta)(1-\chi) G_{\text{adv}} f. \tag{21}$$

The first term on the right-hand side has support in the time slice $\tau_r - \varepsilon < \tau < \tau_r + \varepsilon$. Since $G_{\text{adv}} f$ has support in the past light cone of $\operatorname{supp} f, (1-\chi) G_{\text{adv}} f$ has compact support. Due to the field equation, $\int F \wedge (d+\delta)(1-\chi) G_{\text{adv}} f = 0$; hence the second term does not contribute to the smeared field strength $\int F \wedge f$, and we obtain the identity

右侧第一项的支集在时间切片 $\tau_r - \varepsilon < \tau < \tau_r + \varepsilon$ 上。由于 $G_{\text{adv}} f$ 的支集在 $\operatorname{supp} f, (1-\chi) G_{\text{adv}} f$ 的过去光锥内，故支集是紧的。根据场方程， $\int F \wedge (d+\delta)(1-\chi) G_{\text{adv}} f = 0$ ；因此第二项对弥散场强 $\int F \wedge f$ 没有贡献，我们得到恒等式

$$\int F \wedge f=\int F \wedge (d+\delta)\chi G_{\text{adv}} f$$

which relates the present electromagnetic field with the field at the recombination time.

它将当前的电磁场与复合时期的场联系了起来。

The Maxwell equations are conformally invariant; therefore we may use the codifferential of the conformally transformed metric $\eta + a^{-2}h$. For the unperturbed spacetime we then can use the codifferential δ_0 and the Green operator G^0_{adv} of Minkowski space and get in first order for the perturbed metric $\delta = \delta_0 + \delta_1$ and

麦克斯韦方程具有共形不变性，因此我们可以使用共形变换后度规 $\eta + a^{-2}h$ 的余微分。对于未受扰动的时空，我们可以使用闵可夫斯基空间的余微分 δ_0 和格林算子 G^0_{adv} ，得到扰动度规 $\delta = \delta_0 + \delta_1$ 的一阶结果，且

$$G_{\text{adv}} = G^0_{\text{adv}} - G^0_{\text{adv}} \delta_1 G^0_{\text{adv}} .$$

The measurement of electromagnetic observables related to the radiation from a certain direction then provides information on the metric. The observed correlations between variations of the metric in different directions can now be related to correlation functions of the associated observables in the gravity-dilaton system.

对来自特定方向辐射的电磁可观测量进行测量，就能得到度规的相关信息。不同方向上度规涨落的观测关联，现在可以关联到引力- dilaton 体系中对应可观测量的关联函数。

A rough estimate of the effect can be obtained from the lapse function (cf. (10))

该效应的粗略估计可以通过移滞函数得到 (参见 (10))

$$N = \left|g^{-1}(dT, dT)\right|^{-\frac{1}{2}}.$$

Up to first order it is given by

到一阶近似下，它可表示为

$$N = a\left(1 - \frac{\varphi'}{\phi_0'} - \frac{h_{00}}{2a^2}\right).$$

A gauge-invariant version is

规范不变形式为

$$\tilde{N} = a\left(1 - \frac{\tilde{h}_{00}}{2a^2}\right).$$

The two-point function of $\tilde{h}_{00}$ in an appropriate state of the linearized gravity-dilaton system is then related to the observed variations of the temperature of the CMB

线性化引力-dilaton 体系合适态中 $\tilde{h}_{00}$ 的两点函数，和观测到的 CMB 温度涨落相关联

$$\frac{\delta\mathfrak{T}}{\mathfrak{T}} \approx \frac{\tilde{h}_{00}}{2a^2}.$$

The partial explanation of the observed temperature fluctuations is a strong support for the existence of the dilaton field and represents the up to now only observed effect of the quantization of gravity.

对观测到的温度涨落的部分解释，为 dilaton 场的存在提供了有力支持，也是迄今为止唯一观测到的引力量子化效应。

Concluding Remarks

结束语

We have seen that quantum gravity, in spite of its non-renormalizability, gives rise to a well-defined perturbation series which can be understood as an effective field theory. The problem of the absence of local observables can be treated in terms of relative observables. On generic backgrounds, these relative observables are local functionals of dynamical fields which are used as coordinates. On backgrounds with a high symmetry one has to rely on non-local expressions which will create problems in higher-order perturbation theory.

我们已经看到，量子引力尽管不可重整，却能得到定义良好的微扰级数，可将其理解为有效场论。不存在局部可观测量的问题可以通过相对可观测量来处理。在一般背景下，这些相对可观测量是用作坐标的动力学场的局部泛函。在高对称性背景下，我们必须依赖非局部表达式，这会在高阶微扰理论中引发问题。

We here described a choice of coordinates which are appropriate for FLRW spacetimes. On Minkowski space as a background, one could instead use solutions of the wave equation

我们在此介绍了一种适用于 FLRW 时空的坐标选择。以闵氏空间为背景时，我们也可以改用波动方程的解

$$X^{\mu} = \left(1 - G_{\text{ret}} \Box_{g}\right) x^{\mu}$$

with the canonical coordinates of $\mathbb{R}^{1,3}$. This was used in [20] for a computation of the quantum correction to the Newton formula for the gravitational attraction. This choice, which is related to the harmonic gauge, was also used in several papers of Markus Fröb et al. for more general backgrounds [19]. It avoids some of the pathologies caused by the spatial non-locality of the spatial coordinates introduced in [8].

结合 $\mathbb{R}^{1,3}$ 的正则坐标。文献 [20] 已经使用该方法计算了引力吸引牛顿公式的量子修正。这种和调和规范相关的坐标选择，也被 Markus Fröb 等人的多篇论文用于更一般的背景 [19]。它避免了文献 [8] 中引入的空间坐标所带有的空间非局域性导致的部分病态问题。

In reality, we have all the fields of the standard model which could serve as coordinates, provided we expand around a generic background. So the use of relative observables which are local functionals of local fields is always possible and will yield a well-defined perturbation expansion. If we instead use a highly symmetric background for our convenience, we have to pay for this with the treatment of nonlocal quantities.

实际上，标准模型的所有场都可以用作坐标，只要我们在一般背景附近展开。因此，使用由局部场的局部泛函构造的相对可观测量始终是可行的，并且能得到定义良好的微扰展开。如果我们为了方便转而使用高对称性背景，就必须付出处理非局部量的代价。

Cross-References

交叉引用

Perturbative Approaches to Nonperturbative Quantum Gravity

非微扰量子引力的微扰方法

- Quantum General Relativity and Effective Field Theory

- 量子广义相对论与有效场论

- The Background Information About Perturbative Quantum Gravity

- 微扰量子引力背景知识

- The Functional Renormalization Group in Quantum Gravity

- 量子引力中的函数重整化群

References

参考文献

1. C. Bär, Green-hyperbolic operators on globally hyperbolic spacetimes. Commun. Math. Phys. 333, 1585-1615 (2015)
2. H.-J. Borchers, Über die Mannigfaltigkeit der interpolierenden Felder zu einer kausalen S-Matrix. Il Nuovo Cimento 15, 784-794 (1960)
3. F. Brennecke, M.Dütsch, Removal of violations of the Master Ward Identity in perturbative QFT. Rev. Math. Phys. 20, 119-172 (2008)
4. R. Brunetti, K. Fredenhagen, Microlocal analysis and interacting quantum field theories: renormalization on physical backgrounds. Commun. Math. Phys. 208, 623-661 (2000)
5. R. Brunetti, K. Fredenhagen, R. Verch, The generally covariant locality principle - a new paradigm for local quantum field theory. Commun. Math. Phys. 237, 31-68 (2003)
6. R. Brunetti, M. Dütsch, K. Fredenhagen, Perturbative algebraic quantum field theory and the renormalization groups. Adv. Theor. Math. Phys. 13, 1541-1599 (2009)
7. R. Brunetti, K. Fredenhagen, K. Rejzner, Quantum gravity from the point of view of locally covariant quantum field theory. Commun. Math. Phys. 345(3), 741-779 (2016)
8. R. Brunetti, K. Fredenhagen, T.P. Hack, N. Pinamonti, K. Rejzner, Cosmological perturbation theory and quantum gravity. J. High Energ. Phys. 2016, 32 (2016)
9. R. Brunetti, K. Fredenhagen, P.L. Ribeiro, Algebraic structure of classical field theory: kinematics and linearized dynamics for real scalar fields. Commun. Math. Phys. 368, 519-584 (2019)
10. S. Dodelson, F. Schmidt, Modern Cosmology, 2nd edn. (Elsevier, Amsterdam, 2020)
11. J.F. Donoghue, Quantum general relativity and effective field theory. arXiv:2211.09902 [hep-th]
12. M. Dütsch, From Classical Field Theory to Perturbative Quantum Field Theory, Progress in Mathematical Physics, vol. 74 (Birkhäuser, Cham (Switzerland), 2019)
13. B. Eltzner, Quantization of perturbations in inflation. arXiv:1302.5358
14. H. Epstein, On The Borchers Class of a Free Field. Il Nuovo Cimento 27, 886-893 (1963)
15. H. Epstein, Y. Glaser, The role of locality in perturbation theory. Ann. Inst. Henri Poincaré- Section A XIX(3), 211 (1973)
16. K. Fredenhagen, R. Haag, On the derivation of Hawking radiation associated with the formation of a black hole. Commun. Math. Phys. 127, 273-284 (1990)
17. K. Fredenhagen, K. Rejzner, Batalin-Vilkovisky formalism in the functional approach to classical field theory. Commun. Math. Phys. 314, 93-127 (2012)
18. K. Fredenhagen, K. Rejzner, Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory. Commun. Math. Phys. 317, 697-725 (2013)

19. M. B. Fröb, Gauge-invariant quantum gravitational corrections to correlation functions. Class. Quant. Grav. 35, 055006 (2018)

20. M.B. Fröb, C. Rein, R. Verch, Graviton correction to the Newtonian potential using invariant observables. J. High Energ. Phys. 2022, 180 (2022)

21. D. Goldberg, The Standard Model in a Nutshell (Princeton University Press, Princeton, 2017)

22. M.H. Goroff, A. Sagnotti, The ultraviolet behavior of Einstein gravity. Nucl. Phys. B 226, 709-736 (1986)

23. R. Haag, N.M. Hugenholtz, M. Winnink, On the equilibrium states in quantum statistical mechanics. Commun. Math. Phys. 5(3), 215-236 (1967)

24. T.P. Hack, Quantization of the linearized Einstein-Klein-Gordon system on arbitrary backgrounds and the special case of perturbations in inflation. Class. Quant. Grav. 31(21), 215004 (2014)

25. S.W. Hawking, Particle creation by black holes. Commun. Math. Phys. 43, 199-220 (1975)

26. S.W. Hawking, Information loss in black holes. Phys. Rev. D 72, 084013 (2005)

27. S. Hollands, R. Wald, Existence of local covariant time ordered products of quantum fields in curved spacetime. Commun. Math. Phys. 231, 309-345 (2001)

28. G. t'Hooft, M.J.G. Veltman, One loop divergencies in the theory of gravitation. Ann. Inst. H. Poincare Phys. Theor. A 20, 69-94 (1974)

29. L. Parker, Particle creation in expanding Universes. Phys. Rev. Lett. 21, 562 (1968)

30. G. Popineau, R. Stora, A pedagogical remark on the main theorem of perturbative renormalization theory. Nucl. Phys. B 912, 70-78 (2016)

31. K. Rejzner, Perturbative Algebraic Quantum Field Theory: An Introduction For Mathematicians, Mathematical Physics Studies (Springer, Cham, 2016)

32. R.K. Sachs, A.M. Wolfe, Perturbations of a cosmological model and angular variations of the microwave background. Astrophys. J. 147, 73 (1967)

33. J.M. Stewart, M. Walker, Perturbations of spacetimes in general relativity. Proc. Roy. Soc. Lond. A 341, 49 (1974)

34. R.F. Streater, A.S. Wightman, PCT, Spin and Statistics, and All That, Princeton Landmarks in Physics (Benjamin, New York, 1964)

35. E.C.G. Stückelberg, A. Petermann, La normalisation des constantes dans la théorie des quanta. Helv. Phys. Acta 26, 499-520 (1953)

36. J. Tambornino, Relational observables in gravity: a review. SIGMA 8, 017 (2012)

37. M.T. Tehrani, J. Zahn, Background independence in gauge theories. Ann. Henri Poincaré 21, 1135-1190 (2020)